Decoherence (and quantum quench): the relationship with excited state quantum phase transitions

J. E. García-Ramos[1], J. M. Arias[2], P. Cejnar[3] J. Dukelsky[4], P. Pérez-Fernández[2], A. Relaño[4]

[1]Departamento de Física Aplicada, Universidad de Huelva, Huelva, Spain
[2]Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, Sevilla, Spain
[3]Institute of Particle and Nuclear Physics, Charles University, Prague, Czech Republic
[4]Instituto de Estructura de la Materia, CSIC, Madrid, Spain

Motivation

Quantum Decoherence

- Quantum optics.
- Quantum information.
- Quantum computing.

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Excited State Quantum Phase Transitions (ESQPT).

Index

- Definition of Quantum Decoherence and Quantum Quench.
- Models to be considered: Lipkin and $SU(1,1)$.
- Computation of Quantum Decoherence and Quantum Quench.
- Influence of a ESQPT on Quantum Decoherence or Quantum Quench.

Definition of Quantum Decoherence

Qualitative definition

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How does Quantum Decoherence work?

The wave function of the system $\mathcal{S}$ and the enviroment $\mathcal{E}$ can be factorized if no interaction exits

$$|\Psi\rangle = |S_0\rangle \otimes |\epsilon_0\rangle$$

The interaction will entangle the eigenfunctions

$$|\Psi\rangle = \sum_n |S_n\rangle \otimes |\epsilon_n\rangle$$

Quantum Decoherence in a two-level system

Let us consider a system $\mathcal{S}$ with only two possible levels $|0\rangle$ and $|1\rangle$ that can interact with the enviroment $\mathcal{E}$. We call this system *qubit*.

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Initial state

If we turn on the interaction at $t = 0$ the initial wave function can be written as:

$$|\Psi_{\mathcal{S}\mathcal{E}}(0)>=(a|0\rangle + b|1\rangle) \otimes |\epsilon(0)\rangle$$

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Initial state

If we turn on the interaction at $t = 0$ the initial wave function can be written as:

$$|\Psi_{\mathcal{SE}}(0) >= (a|0\rangle + b|1\rangle) \otimes |\epsilon(0)\rangle$$

The Hamiltonian of System and Enviroment

$$H_{\mathcal{SE}} = I_{\mathcal{S}} \otimes H_{\mathcal{E}} + |0\rangle\langle 0| \otimes H_{\lambda_0} + |1\rangle\langle 1| \otimes H_{\lambda_1}$$

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The decoherence factor

The reduced density matrix

$$\begin{aligned}
\rho_{\mathcal{S}} &= Tr_{\mathcal{E}}|\Psi_{\mathcal{S}\mathcal{E}}(t)><\Psi_{\mathcal{S}\mathcal{E}}(t)| \\
&= |a|^2|0><0| + ab^*r(t)|0><1| \\
&+ a^*br^*(t)|1><0| + |b|^2|1><1|
\end{aligned}$$

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The decoherence factor or fidelity

Back

The decoherence factor corresponds to the off-diagonal terms of the reduce density matrix. It is the overlap of two enviroment states evolving with different Hamiltonians,

$$|r(t)| = \left|\langle\epsilon(0)|\, e^{-iH_0 t} e^{-iH_1 t}\, |\epsilon(0)\rangle\right|$$

If the enviroment is in its ground state:

$$|r(t)| = \left|\langle g_0|\, e^{-iH_1 t}\, |g_0\rangle\right|$$

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Physical meaning of $r(t)$

Let us assume that we prepare a state of the *system* that corresponds to a given superposition of $|0\rangle$ and $|1\rangle$.

The interaction with the enviroment will tend to destroy the superposition state, i.e. it will try "to make colapse" the system wave function.

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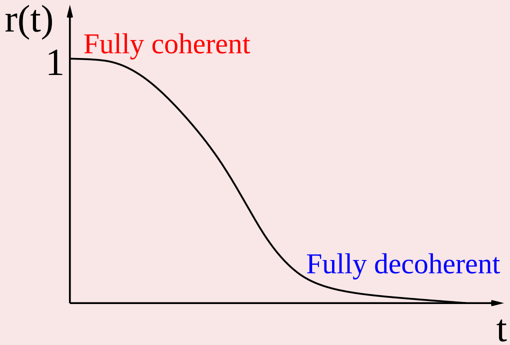
The vanishing of the decoherence factor means that the system is no longer in a superposition state but either in $|0\rangle$ or in $|1\rangle$ with probabilities $|a|^2$ and $|b|^2$ respectively.

Schematic representation



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It is an abrupt change in time of one of the system parameters from an initial value λ to a final one λ'.

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In a closed system, one prepares the ground state $|0_\lambda\rangle$ of the Hamiltonian $H[\lambda]$ and then, it is allowed to evolve with the Hamiltonian $H[\lambda']$. It is therefore a non-equilibrium process.

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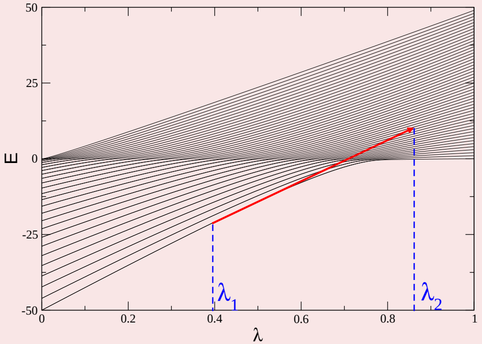

Quantum quench: survival probability

Survival probability, $p_1(t)$

$$p_1(t) = \left|\langle\psi_1|e^{-i\mathcal{H}_2 t}|\psi_1\rangle\right|^2 = \left|\int \underbrace{|\langle\mathcal{E}_2|\psi_1\rangle|^2}_{\omega_1(\mathcal{E}_2)} e^{-i\mathcal{E}_2 t} d\mathcal{E}_2\right|^2$$

$$|\psi_1\rangle = \sum_i \left|\underbrace{\langle\mathcal{E}_{2i}|\psi_1\rangle}_{c_i}\right|^2 |\mathcal{E}_{2i}\rangle$$

$$p_1(t) = \sum_i |c_i|^4 + 2\sum_{i>j} |c_i|^2|c_j|^2 cos[(\mathcal{E}_{2i} - \mathcal{E}_{2i})t]$$

The physical meaning

The survival probability monitors the evolution of $|\psi_1\rangle$ under the Hamiltonian $\mathcal{H}_2$.

See decoherence

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The Hamiltonian for the enviroment in Quantum Decoherence

For a chain of spin with N sites

$$H_{\mathcal{E}} = \alpha \left(\frac{N}{2} + \sum_{i=1}^{N} S_i^z \right) - \frac{4(1-\alpha)}{N} \sum_{i,j=1}^{N} S_i^x S_j^x$$

Schwinger representation

$$H_{\mathcal{E}} = \alpha n_t - \frac{1-\alpha}{N} (Q_t)^2, \quad \text{with} \quad Q_t = s^\dagger t + t^\dagger s + \omega t^\dagger t,$$

where n_t is the number of t bosons and N the total number of bosons.

The qubit and the coupling Hamiltonian

The qubit

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The effective Hamiltonians

$$H_0 = \alpha n_t - \frac{1-\alpha}{N}(Q_t)^2, \quad H_1 = (\alpha + \lambda) n_t - \frac{1-\alpha}{N}(Q_t)^2 .$$

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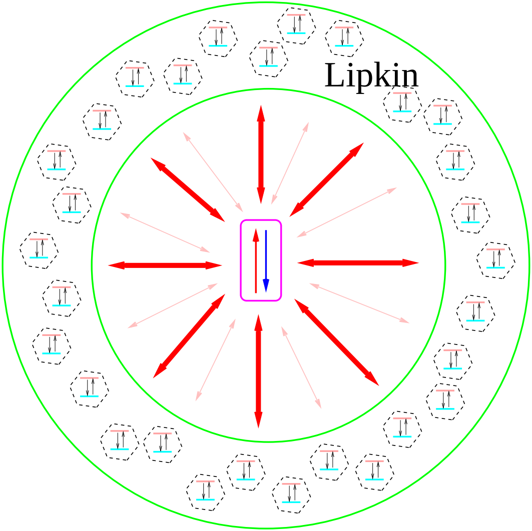
The qubit and the enviroment



The Hamiltonian for the quench: SU(1,1)

For a set of two-atoms molecules that interacts with an ensemble of phonons

$$H^{(1)} = \omega_0 K_0 + \omega b^\dagger b + \frac{\lambda}{\sqrt{M^{(1)}}}\left[bK_+ + b^\dagger K_-\right]$$

where

$$K_+ = \tfrac{1}{2}(a^\dagger)^2,\ K_- = \tfrac{1}{2}\,a^2,\ K_0 = \tfrac{1}{2}\left(a^\dagger a + \tfrac{1}{2}\right)$$

and

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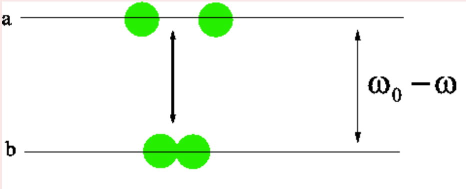

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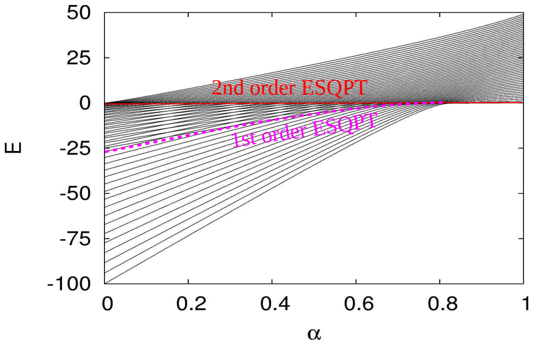
Excite state Quantum Phase Transitions



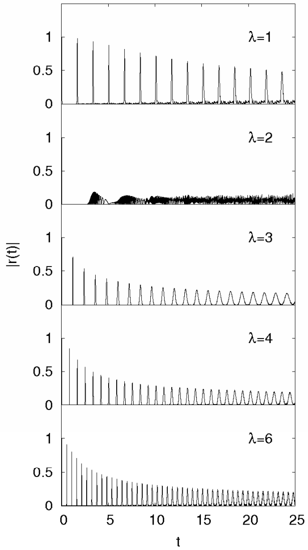
An ESQPT is related with the nonanalitic evolution of certain excited state when varying the control parameter. It is similar to a QPT, but for an excited state. In this case there exists a singularity in the density of states for $E = 0$ and also a region of crossing states.

How to calculate the decoherence factor or the survival probability

To calculate the decoherence factor or the survival probabilty we only need to know the whole set of eigenvalues and eigenfunctions of the Hamiltonian H_1 (perturbed Hamiltonian) and the ground state of the Hamiltonian H_0 (unperturbed Hamiltonian).

$$r(t) \equiv p_1(t) = \sum_k |\langle H_0; GS | H_1; k \rangle|^2 \cdot e^{-\imath E_k^{(H_1)} t}$$

The value of the decoherence factor $|r(t)|$



The decoherence factor, $|r(t)|$, for $\alpha = 0, \omega = 0$ and different values of λ. Note that for $\lambda = 2$, $|r(t)|$ decays almost to zero and then oscillates randomly. The number of bosons is $N = 10000$.

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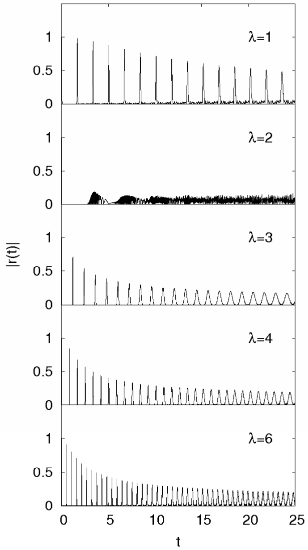

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System plus *enviroment* goes through a QPT at $\lambda_* = 4 - 5\alpha$, for $\alpha < 4/5$.
The almost vanishing of $|r(t)|$ is enhanced for:

$$\lambda_c(\alpha) = \frac{1}{2}(4 - 5\alpha)\,, \quad \alpha < \frac{4}{5}.$$

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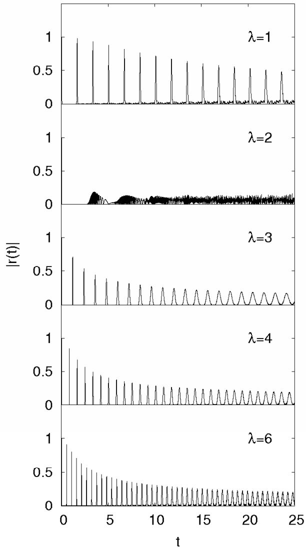

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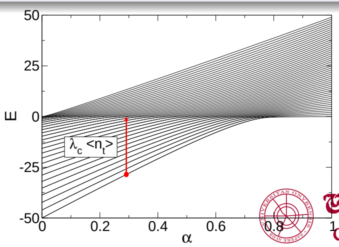

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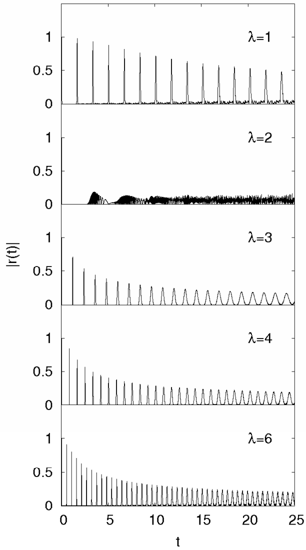
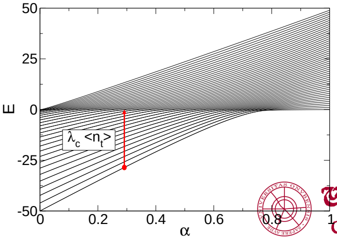


If the system-environment coupling drives the environment to the critical energy E_c of a continuous ESQPT, the Quantum Decoherence induced in the coupled qubit is maximal



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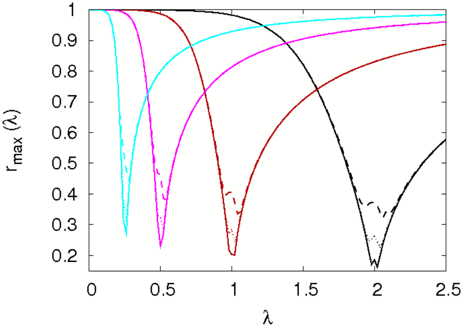
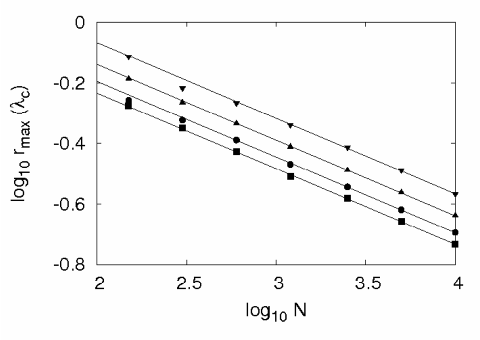
Dependence of r_{max} on λ



Value of $r(t)$ in the second maximum.

- $\alpha = 0$
- $\alpha = 0.4$
- $\alpha = 0.6$
- $\alpha = 0.7$

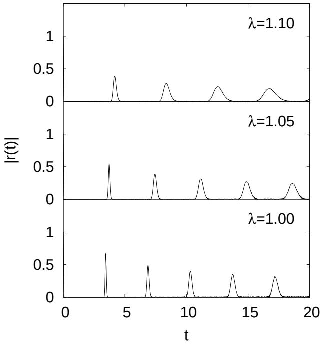
$r_{max}(\lambda_c)$ in the thermodinamical limit

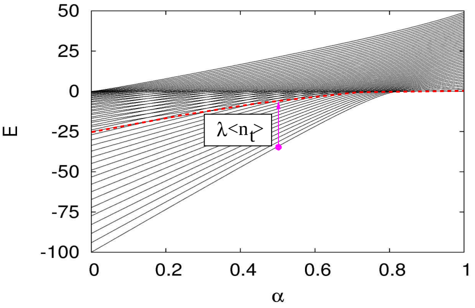


Finite size scaling. The different curves correspond to different values of $\alpha = 0, 0.4, 0.6, 0.7$.

In all the cases we obtain a power law $r_{max}(\lambda_c) \sim N^{-\gamma}$ with $\gamma = 1/4$.

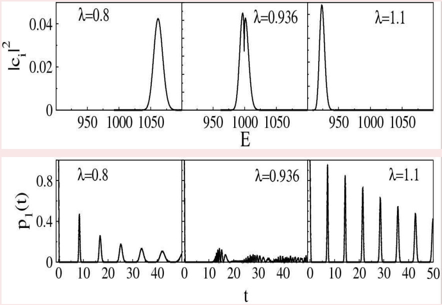
The first order ESQPT

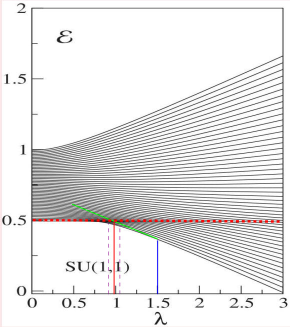




No enhancement of the decoherence is observed ($\omega = 1/\sqrt{2}$).

Survival probability





Conclusions

- We have explored the influence of a ESQPT on the Quantum Decoherence and on the Quantum Quench.
- We have found how the presence of a ESQPT induces the enhancement of the Quantum Decorence and of the Quantum Quench.
- Only second order ESQPT are able to induce the latter behavior.