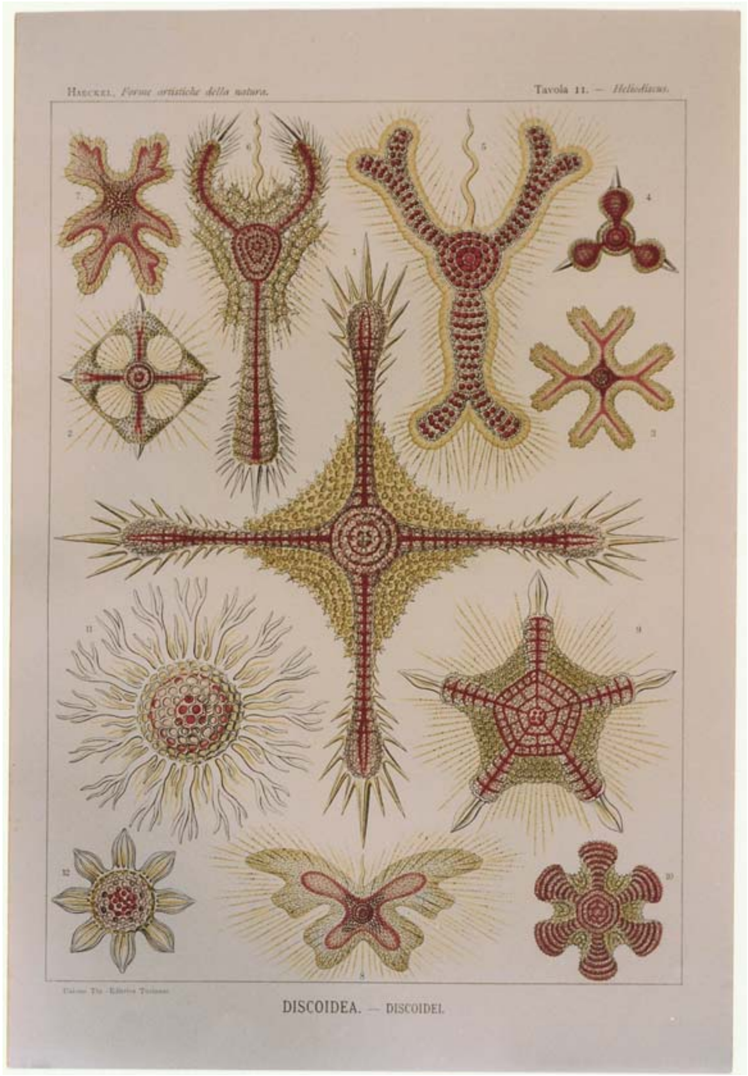
BEAUTY IN NATURE: SYMMETRY

Francesco Iachello
Yale University

Beauty in Physics: Theory and Experiment
Hacienda Cocoyoc, Mexico, May 18, 2012

Many forms of Nature, even the most complex, are often ordered.

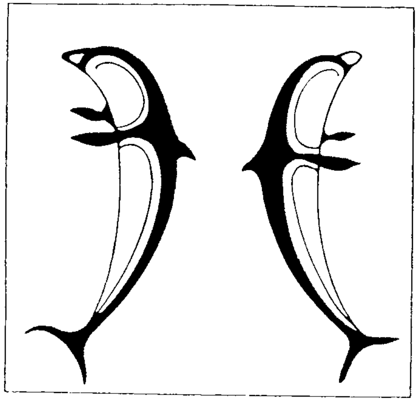


(From E. Haeckel, *Kunstformen der Natur*, Leipzig, 1899.)

Order is synonymous with symmetry:
Greek *συμμετρος*: well-ordered, well-organized

All ancient civilizations attempted to imitate the forms of Nature in Art

Decorative motif
(Sumerian, circa 2000 B.C.)
Translation symmetry

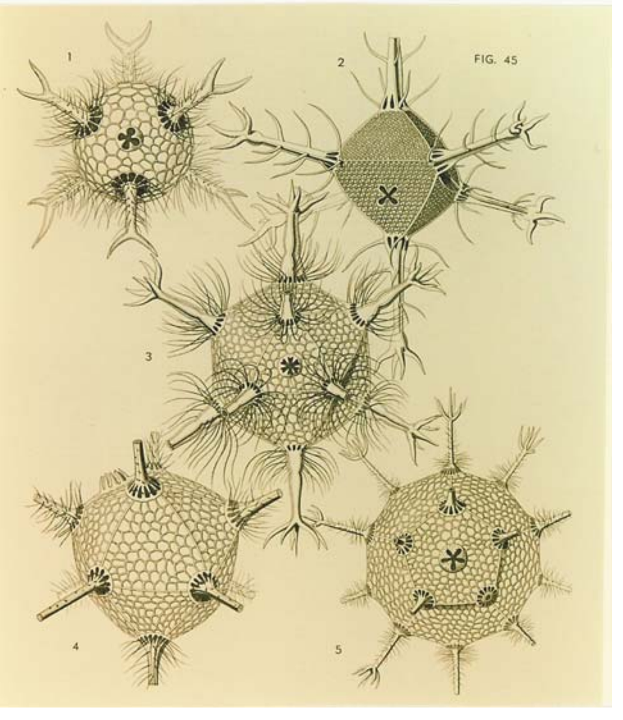


Tile found at the Megaron in Tiryns
(Late Helladic, circa 1200 B.C.)
Reflection symmetry

How to describe symmetry: mathematics.

Greek development of mathematics (geometry): the five regular polyhedra, the tetrahedron, the octahedron, the cube, the icosahedron and the dodecahedron.

Many forms of Nature display polyhedral shapes



(From E. Haeckel, *The Challenger Report*, London, 1887.)

The regular polyedra were associated with the constituents of the Universe: fire (tetra-), air (octa-), earth (cube), water (isosa-) and the Universe itself (pentadodeca-hedron).

The Greeks also thought that symmetry is associated with beauty

Συμμετρος καλος εστιν
[Πολικλειτος, Περι βελοποιικων,IV,2]

(Praying Boy, IV Century B.C.)

[From Hermann Weyl, *Symmetry*, Princeton University Press (1952).]

Esthetic connection!

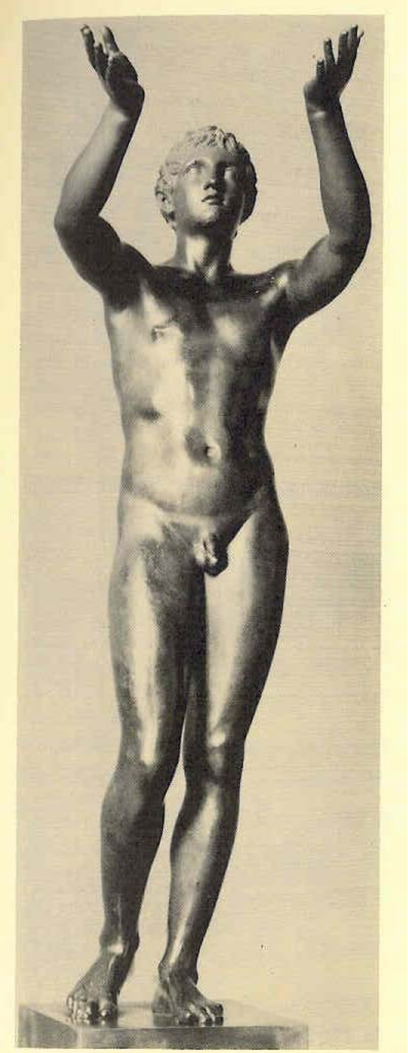


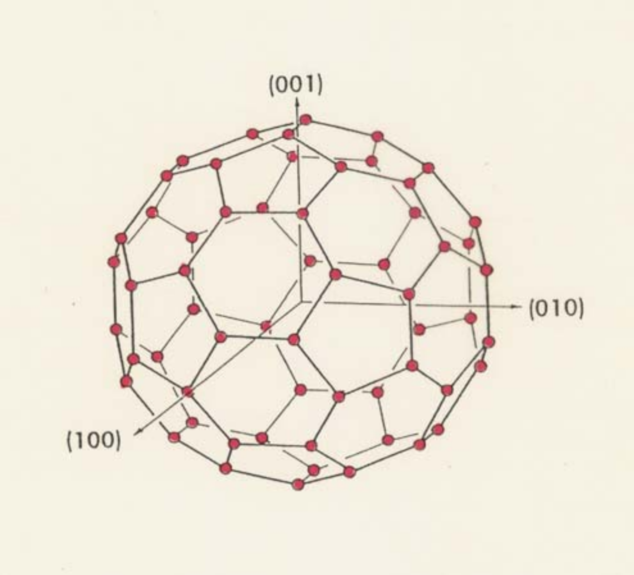
FIG. 2

THE MANY WAYS OF SYMMETRY IN PHYSICS

Symmetry is used today in a variety of ways:

1. Geometric symmetry

Describes the arrangement of constituent particles into a structure
Example: Atoms in a molecule.
Mathematical framework: Point groups



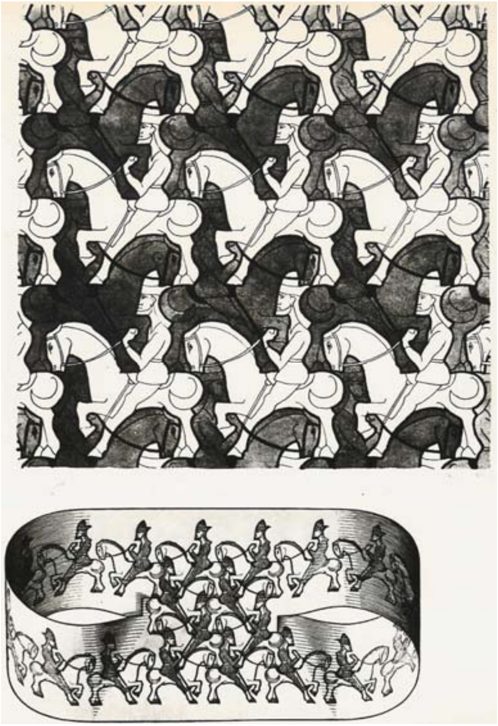
The molecule C_{60} with icosahedral I_h symmetry

(Curl, Kroto and Smalley, 1985)

2. Permutation symmetry

Describes properties of systems of identical particles
Mathematical framework: Permutation group S_n

Became particularly important with the development of quantum mechanics (1920's)



$$\psi\left(1,2\right)=+\psi(2,1)$$ Bosons

$$\psi(1,2)=-\psi\left(2,1\right)$$ Fermions

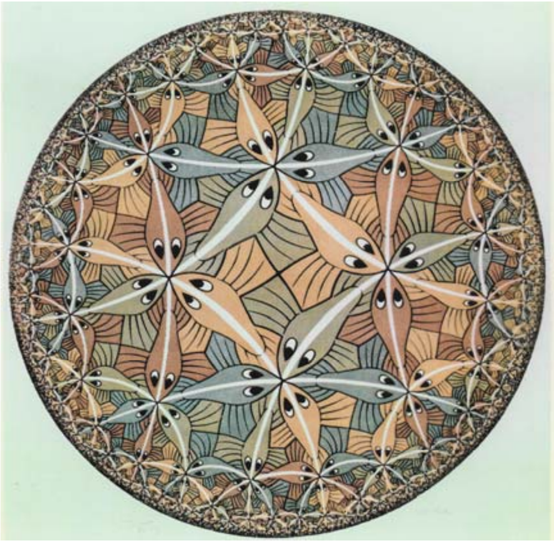
(From M.C. Escher, *Study of the regular division of the plane with horsemen*, 1946)

3. Space-time (or fundamental) symmetry

Fixes the form of the equations of motion.
Mathematical framework: Continuous Lie groups
Example: Free Dirac equation

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x) = 0$$

The free Dirac equation is invariant under the group of Lorentz transformations, SO(3,1), in general under the Poincare' group, ISO(3,1)



(From M.C. Escher, *Circle Limit III*, 1959)

Tessellation of the hyperbolic Poincare' plane

All laws of Nature appear to be invariant under Lorentz transformations!

4. Gauge symmetry

Fixes the form of the interaction between particles and external fields. Fixes the form of the equation satisfied by the fields.
Mathematical framework: Continuous Lie groups
Example: Dirac equation in an external electromagnetic field

$$\left[\gamma_\mu\left(i\partial_\mu - eA_\mu\right) - m\right]\psi(x) = 0$$

The laws of electrodynamics, Maxwell equations, are invariant under U(1) gauge transformations

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

A major discovery of the 2^{nd} part of the 20^{th} Century has been that strong, weak and electromagnetic interactions all appear to be governed by gauge symmetries

$$SU_c(3) \otimes SU_w(2) \otimes U(1)$$

5. Dynamic symmetry

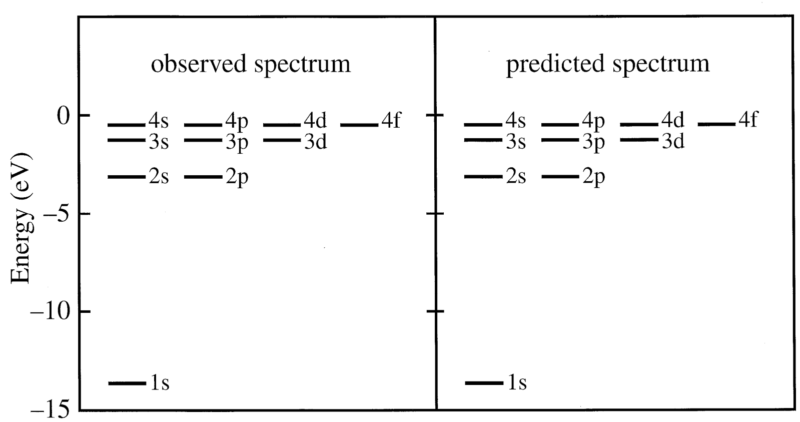
Fixes the interaction between constituent particles and/or external fields. Determines the spectral properties of quantum systems (patterns of energy levels).

Mathematical framework: Continuous Lie groups

Introduced implicitly by Pauli (1926) for the hydrogen atom.

The Hamiltonian with Coulomb interaction is invariant under a set of transformations, G, larger than rotations (Runge-Lenz transformations, SO(4)). It can be written in terms of Casimir operators of G.

$$H = \frac{p^2}{2m} - \frac{e^2}{r} = -\frac{A}{C_2\left(SO(4)\right)+1} \qquad E(n,\ell,m) = -\frac{A}{n^2}$$

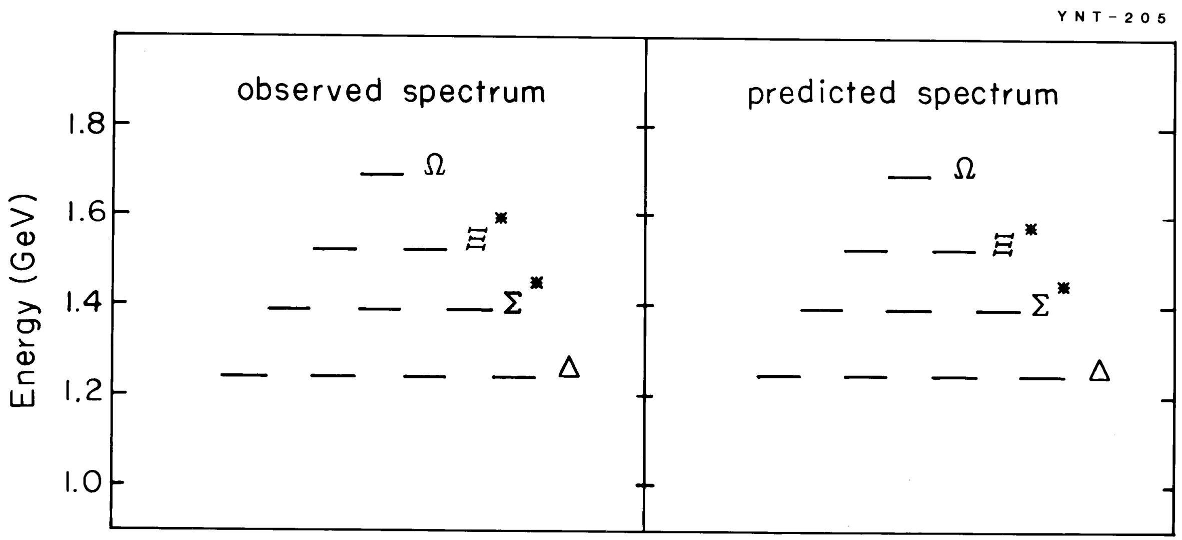


The spectrum of the hydrogen atom

Assumed an important role in physics with the introduction of flavor symmetry (Gell'Mann-Ne'eman, 1962), $SU_f(3)$

$$M = a + b\left[C_1(U(1))\right] + c\left[C_2\left(SU(2)\right) - \frac{1}{4}C_1^2\left(U(1)\right)\right]$$

$$M\left(Y, I, I_3\right) = a + bY + c\left[I(I+1) - \frac{1}{4}Y^2\right]$$



The spectrum of the baryon decuplet is shown as an example of dynamic symmetry in hadrons

A major discovery of the 2nd part of the 20th century has been that dynamic symmetries are pervasive in physics and are found at all scales:

Hadron Physics(GeV) Nuclear Physics(MeV)

Atomic Physics(eV) Molecular Physics(meV)

Example 1: Atomic nuclei (Iachello, 1974; Arima and Iachello, 1976)

Constituents bind in pairs (s and d pairs) treated as bosons (Arima, Otsuka, Iachello and Talmi, 1976):

Interacting Boson Model with algebraic structure u(6)

Dynamic symmetries in this model, obtained by breaking u(6) into its subalgebras

$$u(6) \supset u(5) \supset so(5) \supset so(3) \supset so(2)$$

$$u(6) \supset su(3) \supset so(3) \supset so(2)$$

$$u(6) \supset so(6) \supset so(5) \supset so(3) \supset so(2)$$

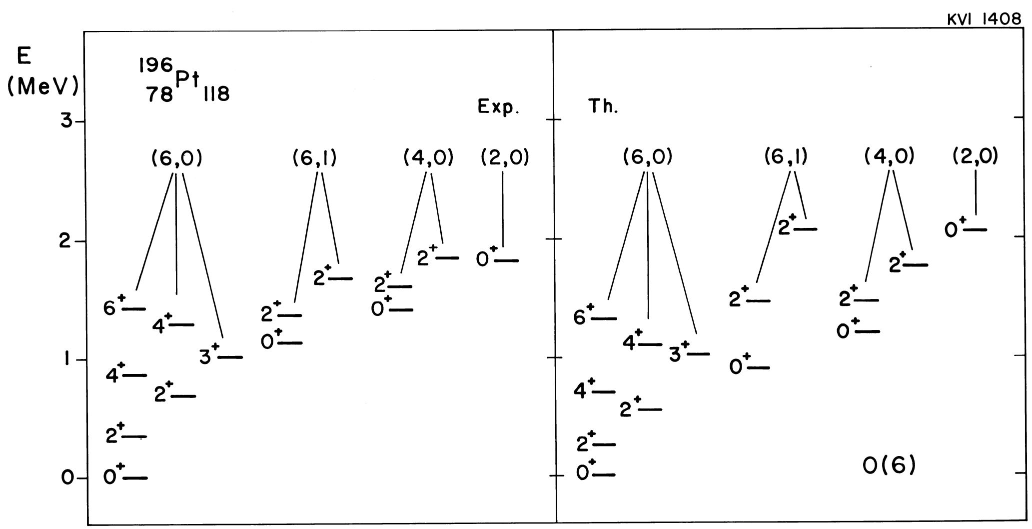
When a dynamic symmetry occurs, all properties can be calculated in explicit analytic form. In particular, the energies of the states are given in terms of quantum numbers.

$$E^{(I)}(N, n_d, v, n_\Delta, L, M_L) = E_0 + \varepsilon n_d + \alpha n_d(n_d+1) + \beta v(v+3) + \gamma L(L+1)$$

$$E^{(II)}(N, \lambda, \mu, K, L, M_L) = E_0 + \kappa(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu) + \kappa' L(L+1)$$

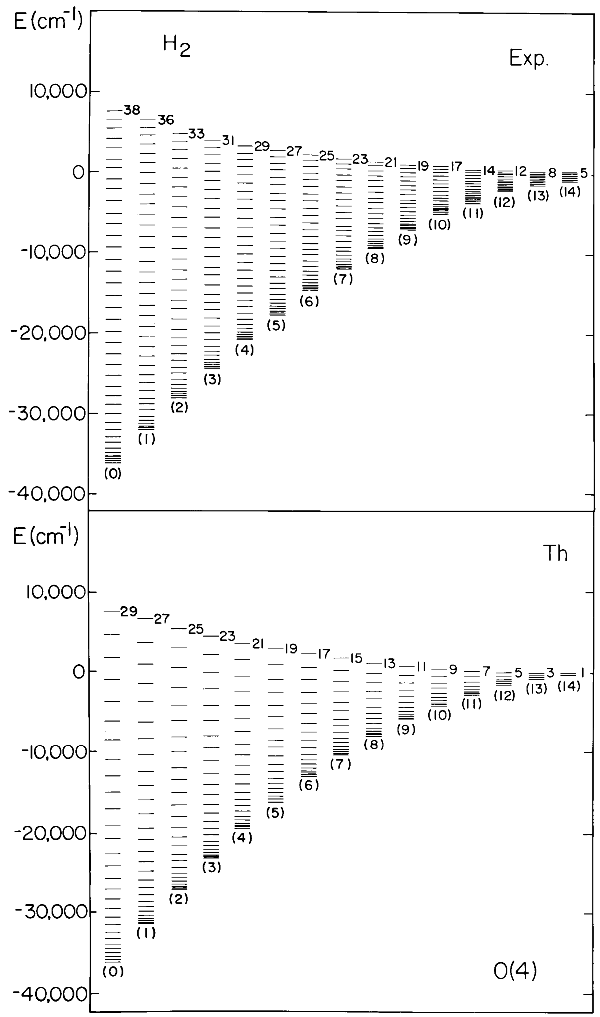
$$E^{(III)}(N, \sigma, \tau, \nu_\Delta, L, M_L) = E_0 + A\sigma(\sigma+4) + B\tau(\tau+3) + CL(L+1)$$

where the various terms are the eigenvalues of the Casimir operators in the appropriate irreducible representations. In the last 30 years, many examples of dynamic symmetries in nuclei have been found.



The spectrum of ^{196}Pt is shown as an example of dynamic symmetry in nuclei (Cizewski, Casten *et al.*, 1978)

Example 2. Molecules (Iachello, 1981). Vibrations and rotations are described in terms of vibrons. Vibron Model with algebraic structure U(4).



The spectrum of H_2 is shown as an example of dynamic symmetry in molecules

SUPERSYMMETRY IN PHYSICS

In the 1970's, in an attempt to further unify the laws of physics, a new concept was introduced: supersymmetry (Volkov and Akulov, 1973; Wess and Zumino, 1974).

Permutation symmetry: bosons and fermions

Discussed previously: systems of bosons or systems of fermions.

Symmetry operations change bosons into bosons or fermions into fermions.

Supersymmetry: symmetry operations change also bosons into fermions and viceversa (appropriate for mixed systems of bosons and fermions).

(From M.C. Escher, *Fish*, circa 1942)

Supersymmetry and its language, Graded Lie algebras and groups, is used today in a variety of ways.

SOME OF THE WAYS OF SUPERSYMMETRY

1. Space-time (fundamental) supersymmetry

A generalization of Lorentz-Poincare' symmetry

Space-time coordinates x,t (bosonic)
Super space-time coordinates θ (fermionic) (Grassmann variables)

Transformations mix x,t and θ

Mathematical framework: SuperPoincare' group

Consequences of supersymmetry: To each particle there corresponds a superparticle (quarks-squarks, etc.)

2. Gauge supersymmetry

Fixes the form of the equations satisfied by the fields
Example: Wess-Zumino Lagrangean (1974)

$$L = L_B + L_F + L_{BF}$$

$$L_B = -\frac{1}{2}\left(\partial_\mu A(x)\right)^2 - \frac{1}{2}\left(\partial_\mu B(x)\right)^2 - \frac{1}{2}m^2 A^2(x) - \frac{1}{2}m^2 B^2(x)$$

$$-gmA(x)\left[A^2(x) + B^2(x)\right] - \frac{1}{2}g^2\left[A^2(x) + B^2(x)\right]$$

$$L_F = -\frac{1}{2}i\overline{\psi}(x)\gamma^\mu\partial_\mu\psi(x) - \frac{1}{2}im\overline{\psi}(x)\psi(x)$$

$$L_{BF} = -ig\overline{\psi}(x)\left[A(x) - \gamma_5 B(x)\right]\psi(x)$$

To each bosonic field there corresponds a fermionic field. Example: Gluons and gluinos

3. Dynamic supersymmetry

Fixes the boson-boson, fermion-fermion and boson-fermion interactions in a mixed system of bosons and fermions
Determines spectral properties of mixed systems of bosons and fermions

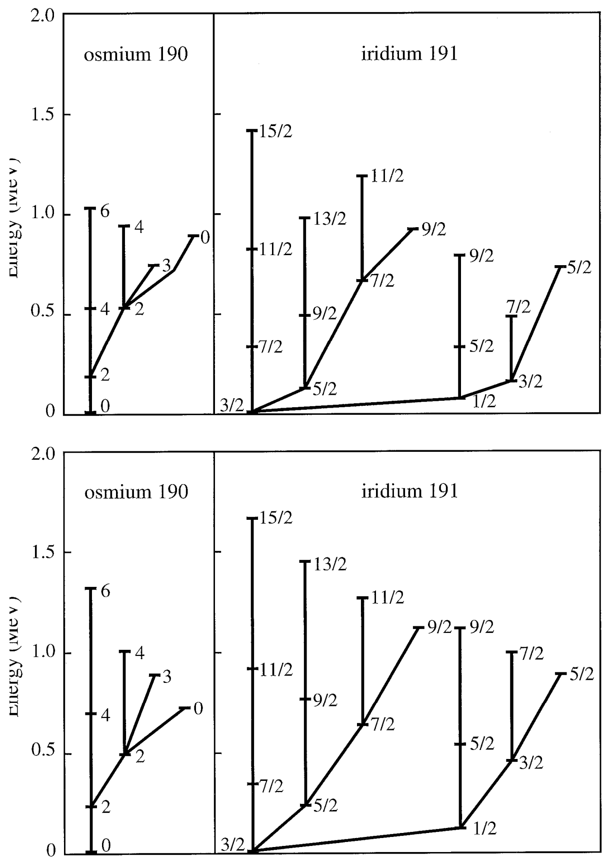
The Hamiltonian $$H = H_B + H_F + V_{BF}$$

is invariant under Bose-Fermi transformations.

Example: Atomic nuclei (Iachello, 1980; Balantekin, Bars, Iachello, 1980; Balantekin, Bars, Bijker, Iachello, 1983; Jolie, Heyde, van Isacker and Frank, 1987)
Some of the constituent bind in pairs (s and d pairs, bosons) while others remain unpaired (fermions): Interacting Boson-Fermion Model with algebraic structure u(6/Ω)

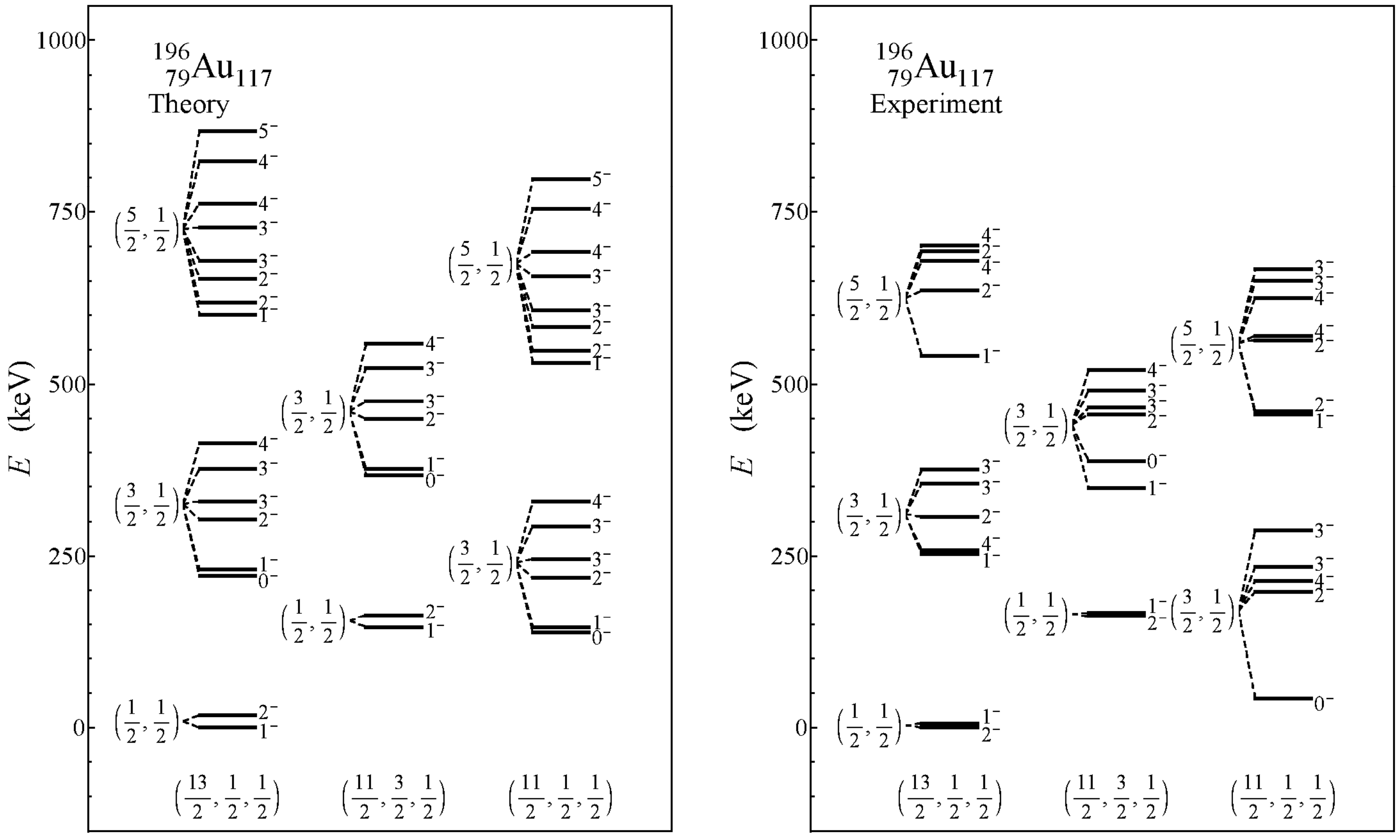
A consequence of dynamic supersymmetry is that all properties of a mixed system of boson and fermions can be calculated in explicit analytic form, and all states can be classified in a given representation of a supergroup.

Dynamic supersymmetries in the Interacting Boson-Fermion Model obtained by breaking u(6/Ω) into its subalgebras (graded or not).



Spectra of Osmium and Iridium nuclei are shown as an example of U(6/4) supersymmetry in nuclei

Dynamic supersymmetry in nuclei, discovered in 1980, has been confirmed recently in a series of experiments involving several laboratories worldwide, especially the Ludwig Maximilian Universität in München, Germany (Graw *et al.*, 2000).



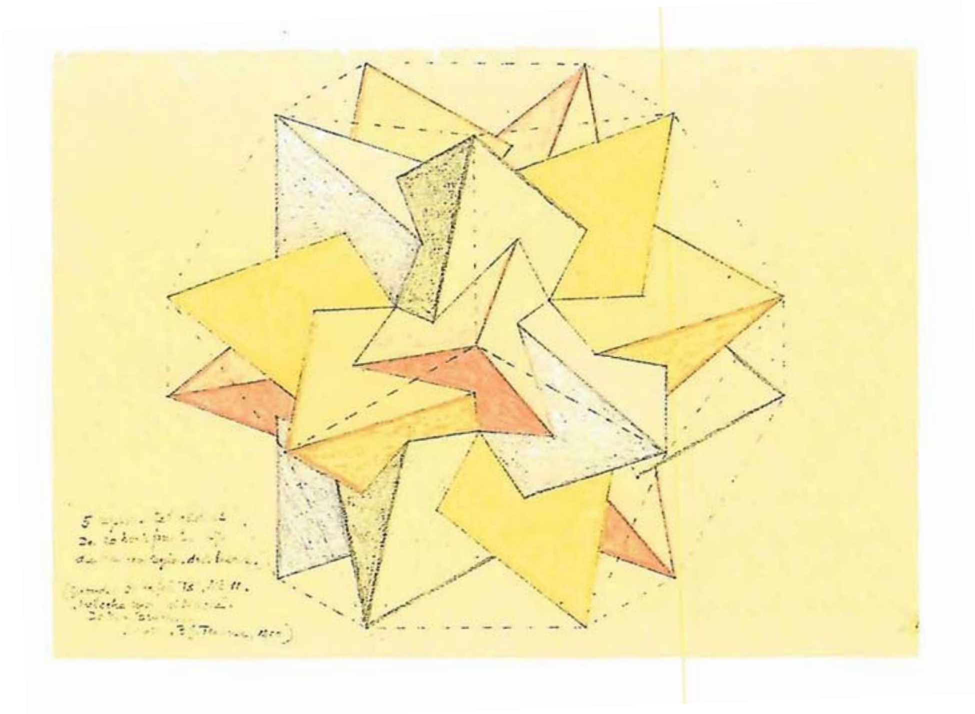
The only experimentally confirmed example of supersymmetry in physics!

CONCLUSIONS

Symmetry=Beauty in its various forms has become a guiding principle in the description of Physics.
[Dirac: If it is beautiful it must be true].
The 20th Century has seen the development of space-time and gauge symmetries as a tool in determining the fundamental laws of physics.
It has also seen the emergence of dynamic symmetry (and supersymmetry) as a way to classify the structure of physical systems.
The 20th Century has also seen the development of new mathematical tools needed to describe symmetries (and supersymmetries).
[Galileo: The book of Nature is written in the language of Mathematics].

In the 21st Century, as the complexity of the phenomena that we are studying increases, symmetry may play an equally important role.

In fact, one of the lessons we have learned is that the more complex the structure, the more useful is the concept of symmetry.



5 regular tetrahedra whose 20 vertices are those of a regular dodecahedron (From M.C. Escher, 1950)

I am therefore looking forward to many more years of application of symmetry concepts to physics and to the development of beautiful models based on these concepts.

I wish to thank all of you present here and those who are not present but have contributed to the study of symmetries in Science for your contributions.
The discoveries mentioned here and others stimulated by these (partial dynamic symmetries, critical symmetries, critical supersymmetries, …; symmetries in multi-fluid systems, proton-neutron interacting boson model, coupled vibron models, …) have been truly collective and without you would not have been possible!

A special thank you goes to Roelof Bijker and Alejandro Frank for organizing this Workshop where many recent developments in the study of symmetry in Science have been presented.