Spherical to Deformed Shape Transitions in the Nucleon Pair Shell Model

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Outline

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- -Ingredients of the Nucleon Pair Shell Model (NPSM)
- -A new approach to the structure of the collective pairs of the NPSM
- -Application to the even Ce isotopes
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- -Summary and closing remarks

Motivation

- Atomic nuclei exhibit a variety of competing collective structures, with phase transitions linking them. Discussed extensively in Interacting Boson Model (IBM), with vertices of Casten triangle representing extreme modes of collective motion.

- Long history of efforts to microscopically derive IBM in various collective regimes. Only recently has it proven possible (Otsuka et al) to provide a unified microscopic derivation, through use of procedure that maps the physics of density functionals directly onto the parameters of the IBM in the various collective regimes.

- We are interested in seeing whether it is possible to describe these competing collective modes using the Nucleon Pair Shell Model (NPSM), a model that like the IBM is based on collective pairs, but in which they are never bosonized.

- Earlier work [Y. Luo, Y. Zhang, X. Meng, F. Pan and J. Draayer , Phys. Rev. C 80 (2009) 014301] suggested that NPSM involving an S pair and a D pair can in principle describe all phase transitions of IBM microscopically. However, these works were of a model nature, never treating real nuclear systems.
- Here, we'll consider *real* nuclei that undergo a shape transition from vibrational to rotational character to see whether the NPSM can describe the observed shape transition.

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The Nucleon Pair Shell Model

-The NPSM is a truncation scheme based on the inclusion of a set of collective pairs of alike nucleons.

-Has its origin in the Generalized Seniority or Broken Pair Approximations.

-Became a practical approach for treating systems with many pairs through the development by Chen [Nucl. Phys. A626 (1997) 686] of a recursive formula that enables treatment of systems with *many* non-S pairs.

-Several applications of the method have been reported for nuclei in a variety of nuclear regimes.

-Review article now in preparation by Y.M. Zhao and A. Arima.

Key ingredients of the NPSM

-Requires a model space and an effective shell-model hamiltonian. Computational limitations of methodology currently restrict hamiltonians to include multipole pairing interactions in *n-n* and *p-p* channels and separable multipole interactions in *p-n* channel.

-Requires a set of collective pair degrees of freedom in terms of which model space defined. Typically includes S_ν , S_π , D_ν and D_π , *but can include others as well, either* S, D *or non*-S, D.

-Pairs *typically* chosen using the Generalized Seniority or Broken Pair Approach. S pairs obtained from number-conserving BCS treatment (variational treatment in *w*=0 space) and non-S pairs from variational treatment in *w*=2 space. For D pairs can instead use a commutator treatment.

-Traditional description of collective S and D pairs fine for either semi-magic or near semi-magic nuclei, but not for nuclei with sufficiently large numbers of neutrons and protons, where *p-n* correlations need to be taken into account in defining dominant collective pairs.

-Will consider a method for defining the dominant collective pairs for the NPSM that dynamically incorporates the key physics of the *p-n* interaction.

Collective pairs appropriate for an NPSM description of deformed nuclei

-For well-deformed nuclei, dominant collective pairs can be chosen based on a Hartree Fock Bogolyubov treatment [SP and J. Dukelsky, Phys. Lett. 128B (1983) 9]

-For axially-symmetric nuclei, HFB wave function in canonical basis given by

$$|\Phi> = c \exp\Big(\sum_{\tau=\nu,\pi}\Gamma_{\tau}^{+}\Big)|\tilde{0}>$$

where

$$\Gamma_{\tau}^{+} = \sum_{\mathrm{L\ even}} \mathrm{a}_{\mathrm{L}}^{(\tau)}\, \Gamma_{\tau}^{(\mathrm{L})+}$$

structure of
dominant L pair

amplitude of L pair
in HFB coherent state

Other features of calculation

-*Nuclei:* even mass Ce isotopes, from ^{142}Ce through ^{148}Ce

-*Model space:* assume a Z=50, N=82 core and distribute 8 valence protons over orbits in 50-82 shell and 2-8 valence neutrons over orbits in 82-126 shell.

-*Hamiltonian:*

$$H = \sum_{\tau=\nu,\pi} \left(\sum_{j} \varepsilon_{\tau\,\mathrm{j}} \tau_j^{+} \tau_j - \sum_{\mathrm{L}=0,2} \mathrm{G}_{\tau\,\mathrm{L}}\, \mathrm{A}^{+}_{\tau\,\mathrm{L}} \cdot \mathrm{A}_{\tau\,\mathrm{L}} \right) - \kappa\, Q_{\pi} \cdot Q_{\nu}$$

p-n interaction

single-particle energies

n-n and p-p interactions

Hamiltonian parameters

Single-particle energies – from mass-133 nuclei

	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$g_{7/2}$	$h_{11/2}$
ε_{π} *(MeV)*	2.990	2.708	0.962	0.	2.793

	$p_{1/2}$	$p_{3/2}$	$f_{5/2}$	$f_{7/2}$	$h_{9/2}$	$i_{13/2}$
ε_{ν}*(MeV)*	1.656	0.854	2.005	0.	1.561	1.800

Two-proton interaction – from Binding Energy and 2_1^+ energy of ^{134}Te
$G_{\pi 0}$=-0.18 *MeV* , $G_{\pi 2}$=0.

Two-neutron interaction – from Binding Energy and 2_1^+ energy of ^{134}Sn
$G_{\nu 0}$=-0.13 *MeV* , $G_{\nu 2}$=-0.012 *MeV*

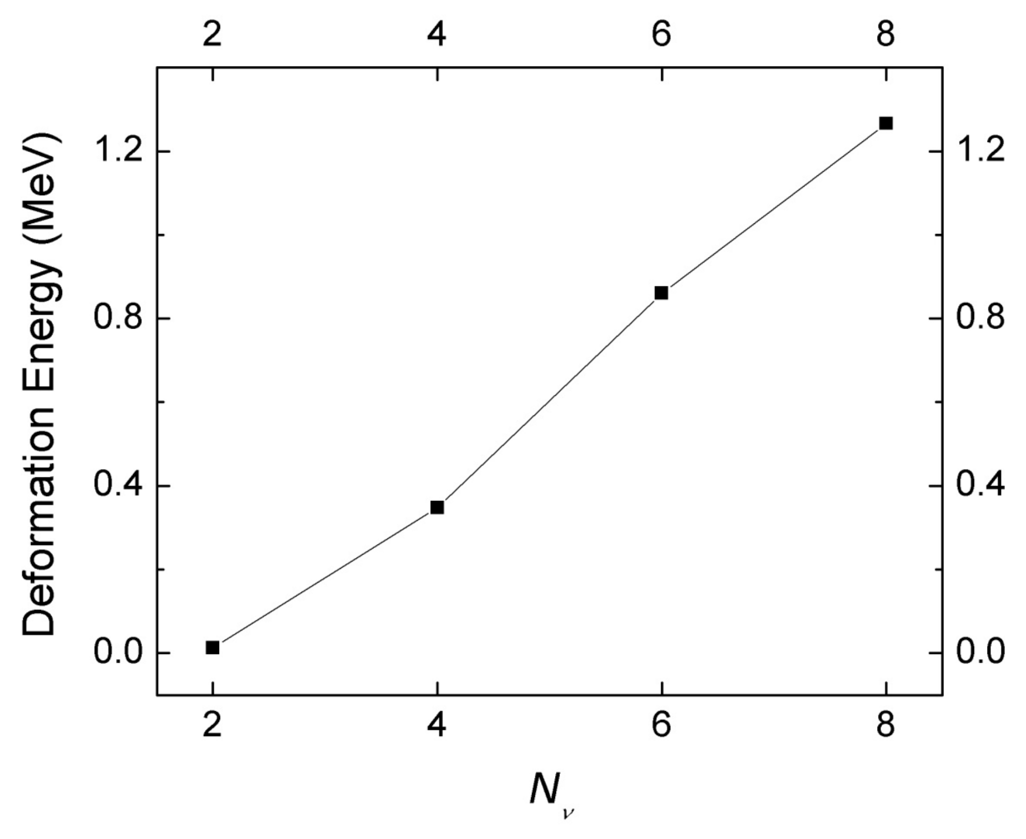
p-n interaction strength - from optimal description of ^{136}Te
κ= -0.20 *MeV*

Question: Do even-mass Ce isotopes exhibit vibrational to rotational shape transition?

	^{142}Ce	^{144}Ce	^{146}Ce	^{148}Ce	SU(3)
$R_{4/2}$	1.90	2.36	2.58	2.87	3.33
$R_{6/2}$	2.72	4.15	4.52	5.32	7.00

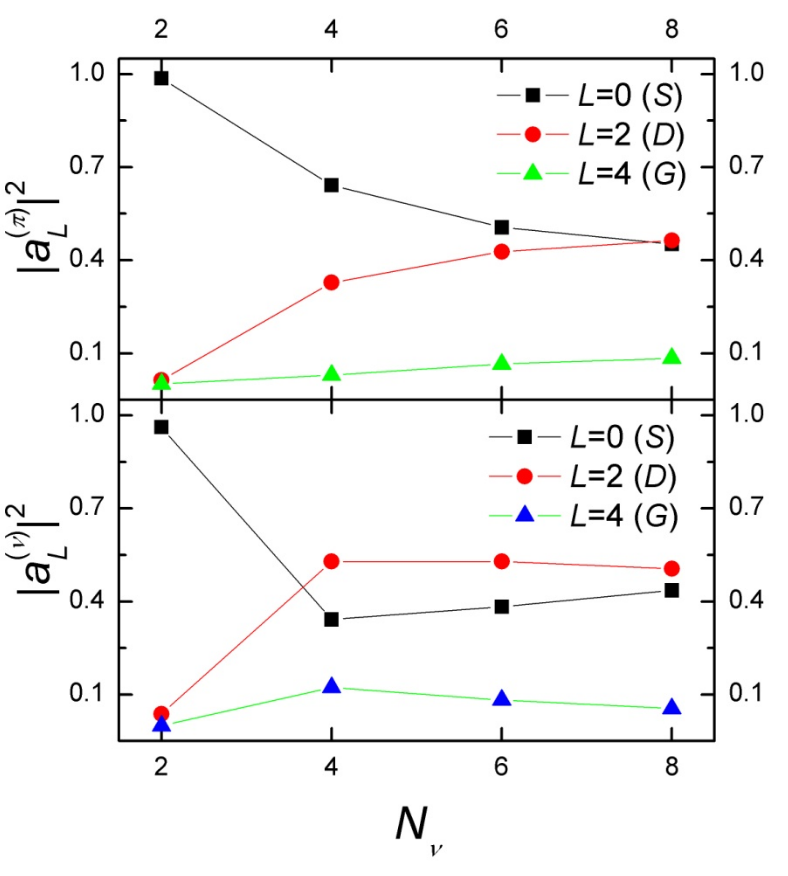
Answer: Yes!

HFB deformation energies



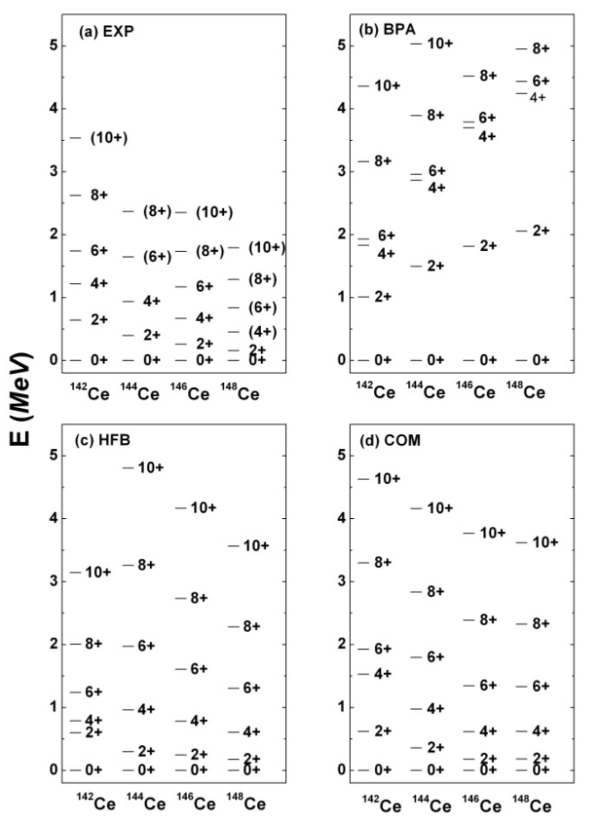
Deformation energy grows gradually with increasing neutron number, though not to as large a value as we would have liked.

Dominant pairs from HFB



In deformed regime, comparable number of S and D pairs in condensate.

Results from NPSM calculations



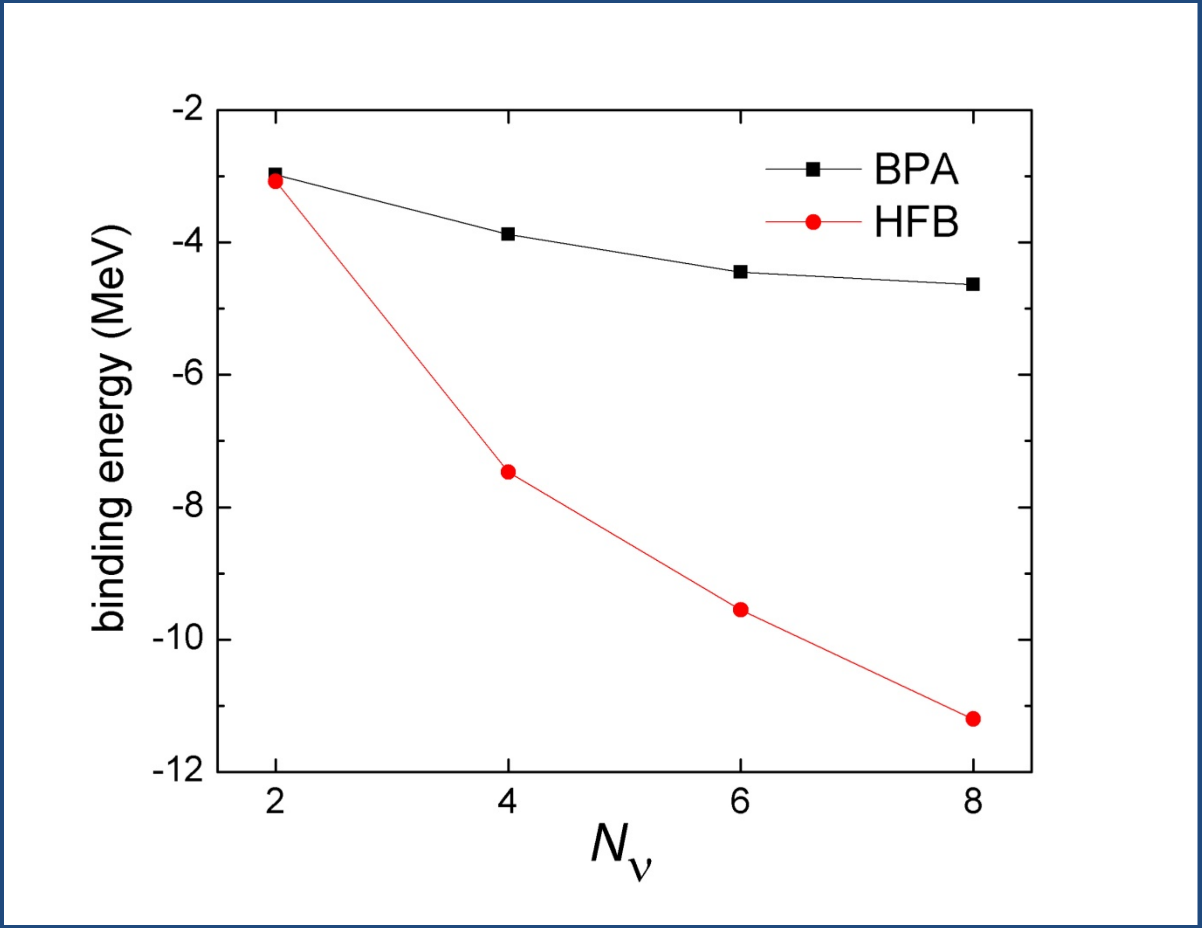
-BPA uses traditional pairs from Broken Pair Approximation. Does not produce shape transition.

-HFB uses pairs based on HFB Approximation. Does produce shape transition. But much too rapid.

-COM includes both types of pairs. Improves description of all isotopes considered.

-None give large enough moment of inertia for most deformed system.

Binding energies from different NPSM calculations



HFB pairs give much more binding than BPA pairs, since they include more correlations.

Summary and future outlook

-The Nucleon Pair Shell Model cannot describe a shape transition from spherical to deformed nuclei in real nuclei when it uses the traditional Broken Pair approach to define the correlated pairs of the model.

-If we use a variational prescription that incorporates *p-n* correlations, e.g. projection from a HFB coherent state, we can produce a shape transition. However, the shape transition is too rapid. Furthermore, we can not obtain a large enough moment of inertia in the deformed region.

-Results can be improved by including more than one collective pair with each angular momentum, but thought still needed as to how to optimally choose these pairs in the transitional region.

-The fact that the Nucleon Pair Shell Model works directly in the fermion space makes it possible to include non-collective pair degrees of freedom on the same footing, as could be important in describing backbending phenomena.

Gamma ray energies in ground band