# Beautiful Graphene, Photonic Crystals, Schrödinger and Dirac Billiards and Their Spectral Properties

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Cocoyoc 2012

- Something about graphene and microwave billiards
- Dirac spectrum in a photonic crystal
    - Experimental setup
    - Transmission and reflection spectra
- Photonic crystal in a box: Dirac billiards
    - Measured spectra
    - Density of states
    - Spectral properties
- Outlook

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S. Bittner, B. Dietz, J.Isensee, T. Klaus, M. Miski-Oglu, A. R., C. Ripp
N.Pietralla, L. von Smekal, J. Wambach



ECT*

Institut für Kernphysik S-DALINAC Technische Universität Darmstadt

# Nobel Prize in Physics 2010



Photo: Sergeom, Wikimedia Commons

## Andre Geim


Photo: University of Manchester, UK

## Konstantin Novoselov

The Nobel Prize in Physics 2010 was awarded jointly to Andre Geim and Konstantin Novoselov *"for groundbreaking experiments regarding the two-dimensional material graphene"*

# Graphene

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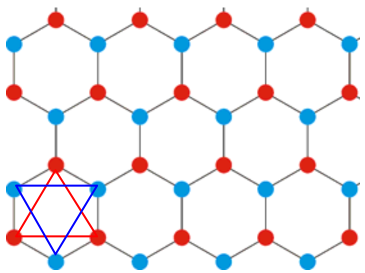
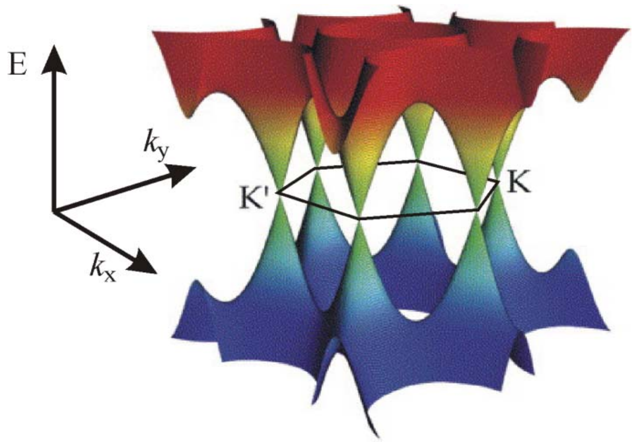
- "What makes graphene so attractive for research is that the spectrum closely resembles the Dirac spectrum for massless fermions."
  M. Katsnelson, Materials Today, 2007

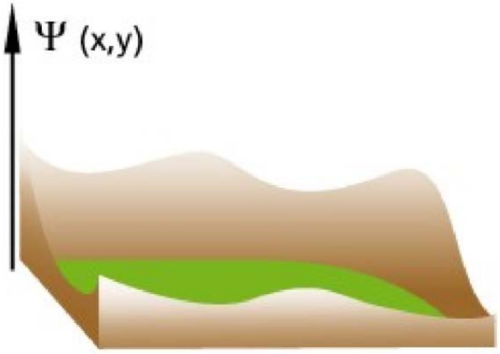




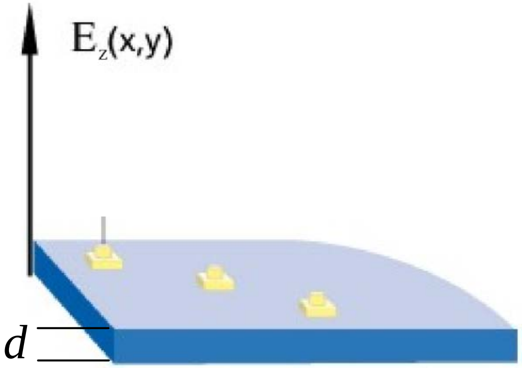

- Two triangular sublattices of carbon atoms
- Near each corner of the first hexagonal Brillouin zone the electron energy $E$ has a conical dependence on the quasimomentum
- $E = \hbar v_F k$ but low $v_F \approx c/300$
- Experimental realization of graphene in analog experiments of microwave photonic crystals

# Quantum Billiards and Microwave Billiards

Quantum billiard




Microwave billiard




Analogy for $\lambda > 2d$

$$\left(\frac{\hbar}{2m}\Delta + E\right)\Psi = 0, \quad \Psi|_{\partial\Omega} = 0 \quad \longleftrightarrow \quad \left(\Delta + k^2\right) E_z = 0, \quad E_z|_{\partial\Omega} = 0$$

eigenvalue $E$ $\leftrightarrow$ wave number $k = \frac{2\pi f}{c}$

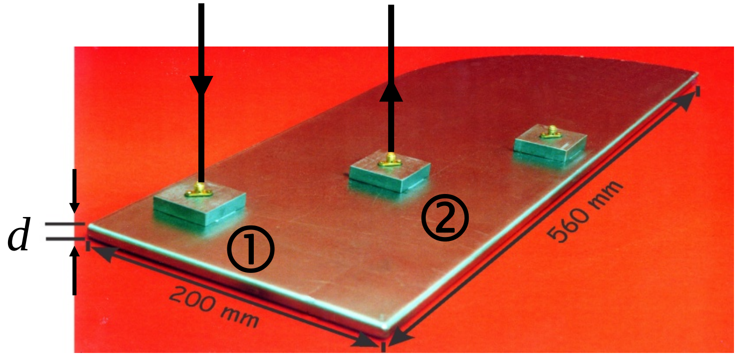
eigenfunction $\Psi$ $\leftrightarrow$ electric field strength $E_z$

# Measurement Principle

- Measurement of scattering matrix element $S_{21}$

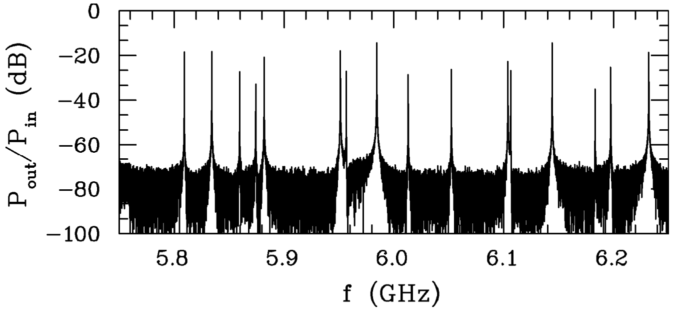
rf power in

rf power out




$$\frac{P_{out,2}}{P_{in,1}} = |S_{21}|^2$$

Resonance spectrum




Resonance density

$$\rho(f) = \sum_{\nu} \delta(f - f_{\nu})$$

$$\rho(f) = \rho_{Weyl}(f) + \rho_{fluc}(f)$$

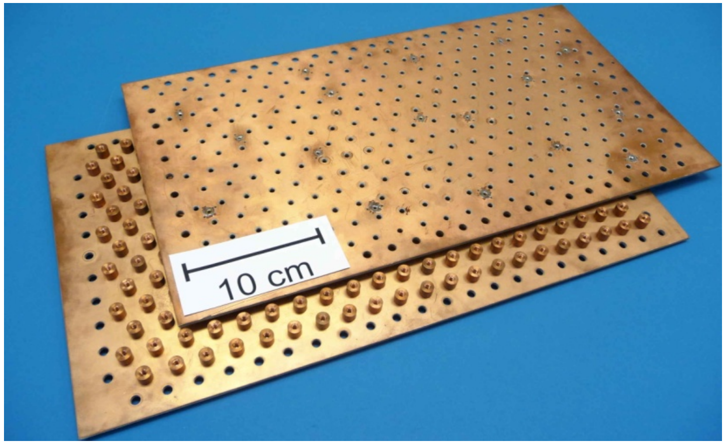
FT

Length spectrum (Gutzwiller)

$$\tilde{\rho}(l)$$

# Open Flat Microwave Billiard: Photonic Crystal

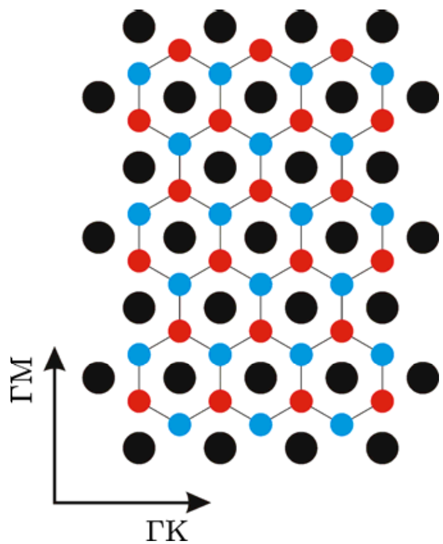
- A photonic crystal is a structure, whose electromagnetic properties vary periodically in space, e.g. an array of metallic cylinders
  → open microwave resonator

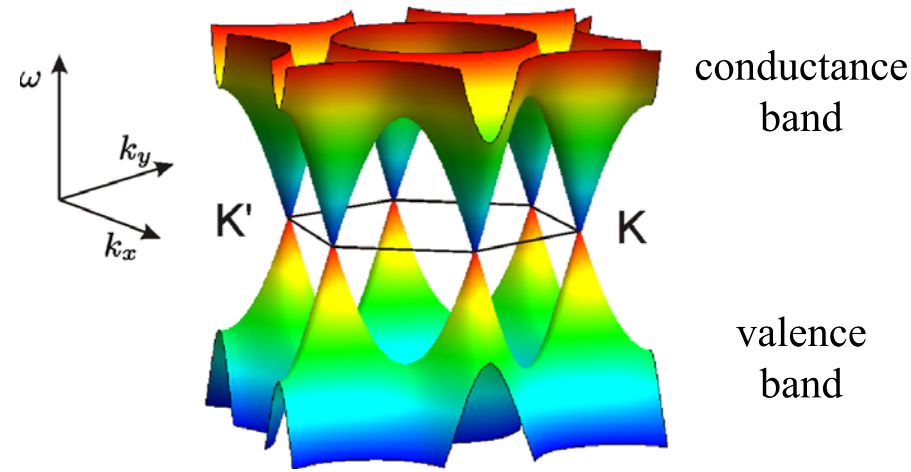



- Flat "crystal" (resonator) → E-field is perpendicular to the plates ($TM_0$ mode)
- Propagating modes are solutions of the scalar Helmholtz equation
  → Schrödinger equation for a quantum multiple-scattering problem
  → Numerical solution yields the band structure

# Calculated Photonic Band Structure

- Dispersion relation $\omega(\vec{k})$ of a photonic crystal exhibits a band structure analogous to the electronic band structure in a solid




- The triangular photonic crystal possesses a conical dispersion relation
  → Dirac spectrum with a Dirac point where bands touch each other
- The voids form a honeycomb lattice like atoms in graphene

# Effective Hamiltonian around the Dirac Point

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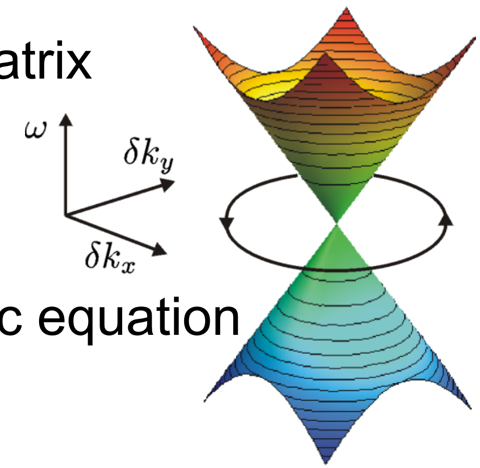
- Close to Dirac point the effective Hamiltonian is a 2x2 matrix

$$\hat{H}_{\text{eff}} = \omega_D \mathbb{1} + v_D \left( \delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y \right)$$

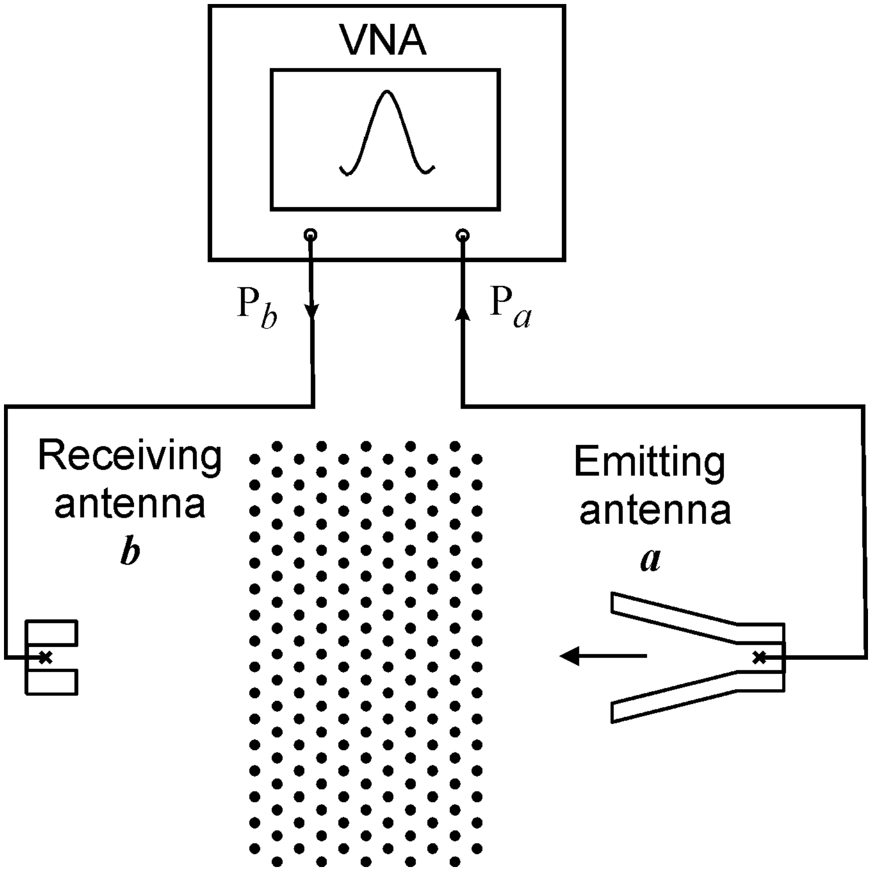



- Substitution $\delta k_x \rightarrow -i\partial_x$ and $\delta k_y \rightarrow -i\partial_y$ leads to the Dirac equation

$$\begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = i \, \frac{\omega - \omega_D}{v_D} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- Experimental observation of a Dirac spectrum in open photonic crystal
  (S. Bittner *et al.*, PRB **82**, 014301 (2010))
- Scattering experiment

# Scattering Experiment




- Horn antenna emits approximately plane waves
- VNA measures the modulus of the scattering matrix given by
  $|S_{ba}|^2 = \frac{P_b}{P_a}$
- Transmission: $|S_{ab}|^2, |S_{ba}|^2$
- Reflection: $|S_{aa}|^2, |S_{bb}|^2$

# Experimental Realization of 2D Photonic Crystal

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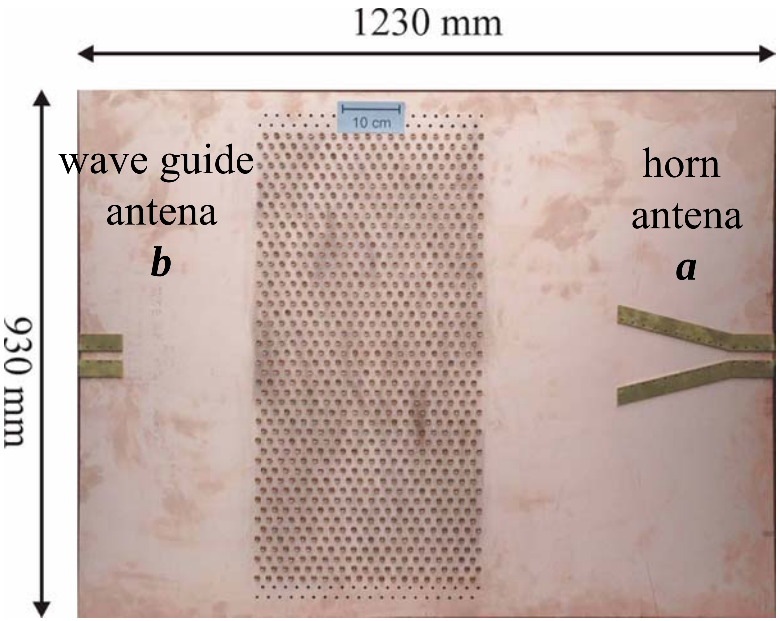
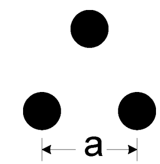


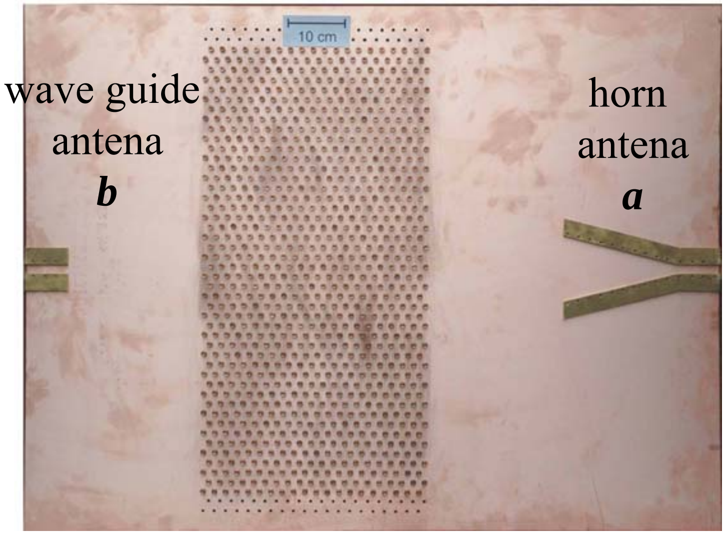

- # cylinders: 23 x 38 = 874
- Cylinder radius: R = 5 mm
- Lattice constant: a = 20 mm

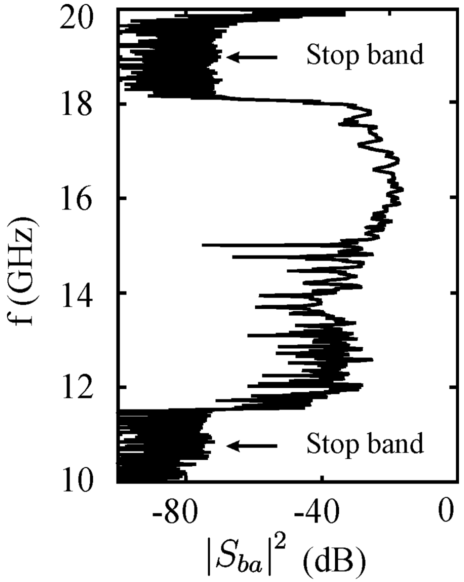



- Crystal size: 400 x 900 x 8 mm
- Frequency: $f_{max}$=19 GHz

- First step: experimental observation of the band structure

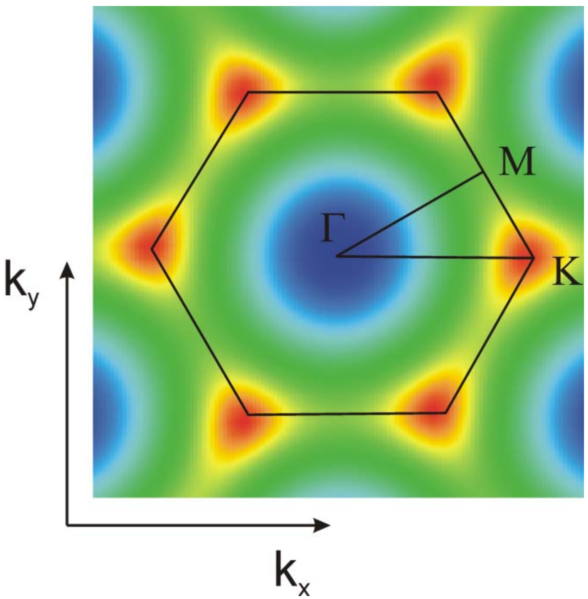
# Transmission through the Photonic Crystal

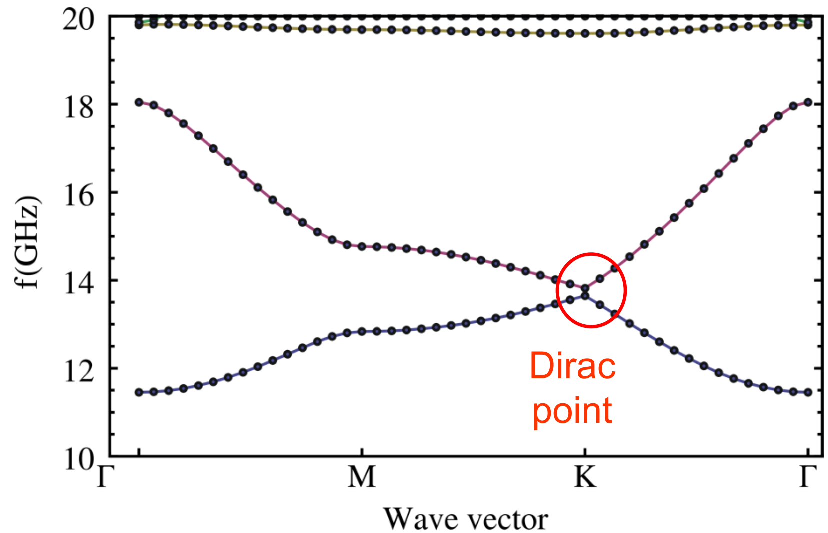



- Transmission spectrum possesses two stop bands
- Comparison with calculated band structure

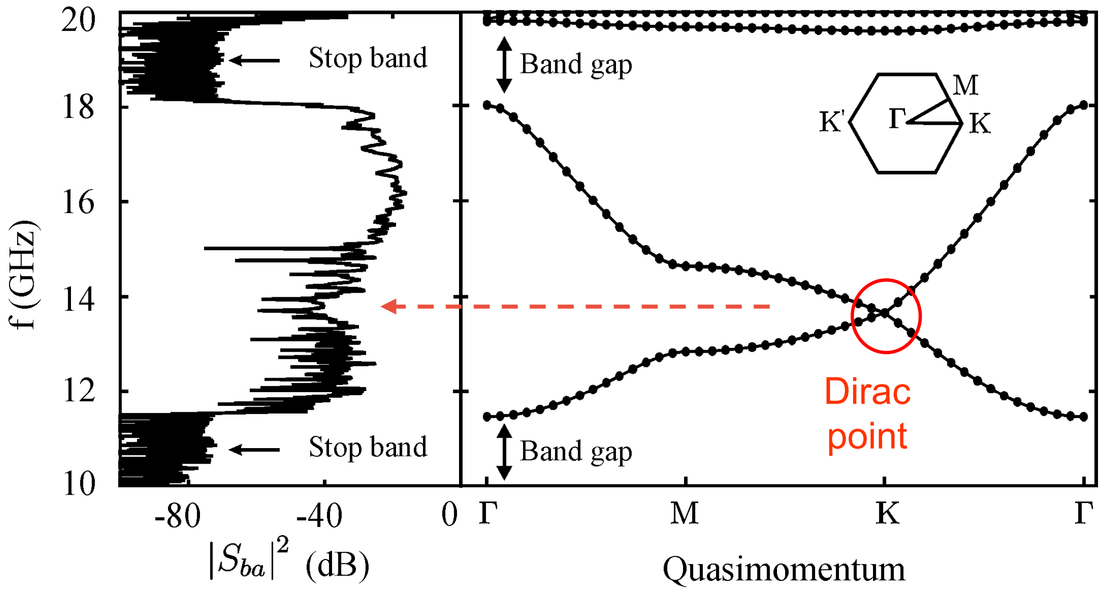
# Projected Band Diagram




- The density plot of the 1st frequency band
- The projected band diagram along the irreducible Brillouin zone ΓMK
- The 1st and 2nd frequency bands touch each other at the corners of the Brillouine zone → Dirac Point

# Transmission through the Photonic Crystal




- The positions of measured stop bands coincide with the calculated ones → lattice parameters chosen correctly
- Dirac point is not sufficiently pronounced in the transmission spectra → single antenna reflection measurement

# Single Antenna Reflection Spectrum

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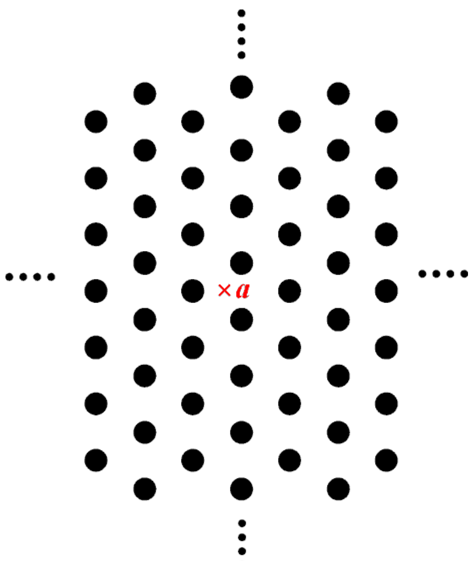
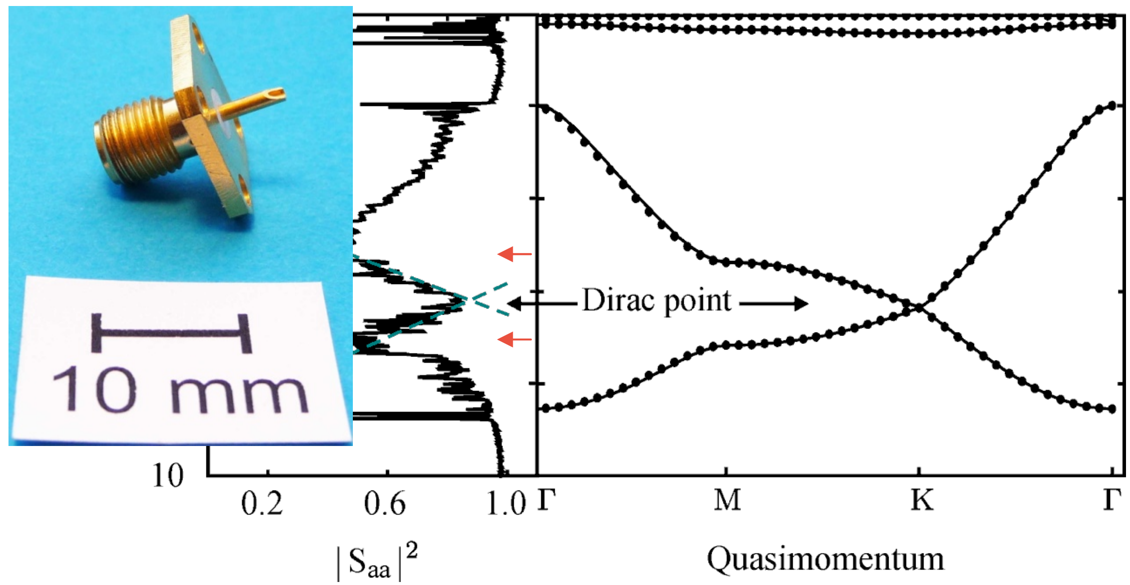
- Measurement with a wire antenna *a* put through a drilling in the top plate → point like field probe




- Characteristic cusp structure around the Dirac frequency
- Van Hove singularities at the band saddle point $|\vec{\nabla}_k\,\omega(\vec{k})| = 0$
- Next: analysis of the measured spectrum

# Local Density of States and Reflection Spectrum

- The scattering matrix formalism relates the reflection spectra to the local density of states (LDOS)

$$1 - |S_{aa}(f)|^2 \propto L(\vec{r}_a, f)$$

- LDOS

$$L(\vec{r}, f) \propto \int_{BZ} |\psi(\vec{k}, \vec{r})|^2 \frac{1}{2\pi} \delta(f - f(\vec{k})) d^2k$$

- LDOS around the Dirac point (Wallace,1947)

$$L(\vec{r}_a, f) \sim \frac{\langle|\psi(\vec{r}_a)|^2\rangle}{v_D^2} |f - f_D|$$

- Three parameter fit formula $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

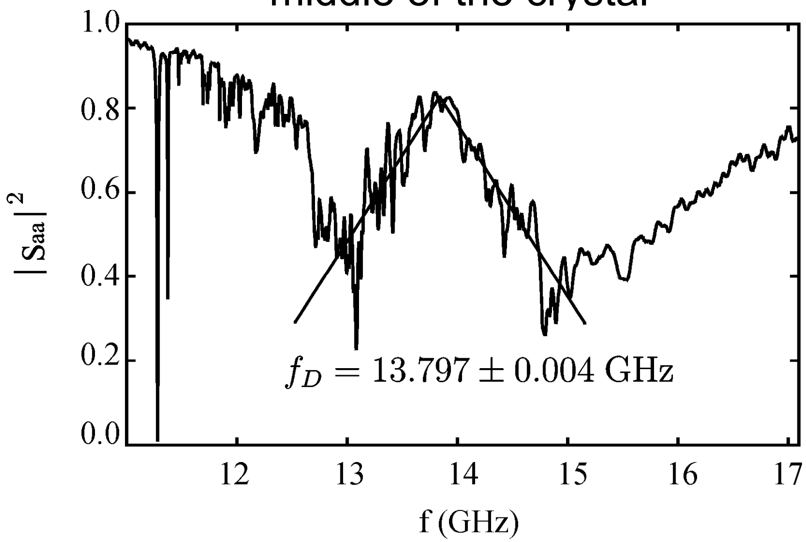
fit parameters ($D$, $C$, $f_D$)

# Reflection Spectra

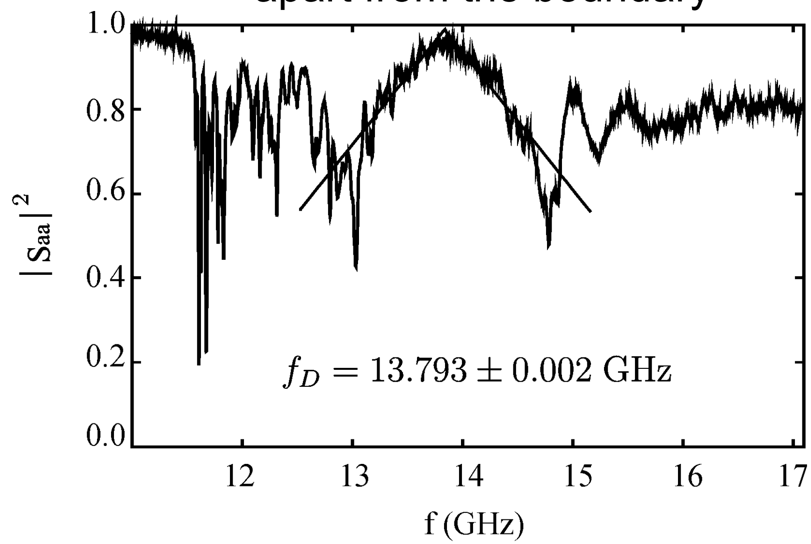


- Description of experimental reflection spectra $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

antenna ***a*** in the middle of the crystal



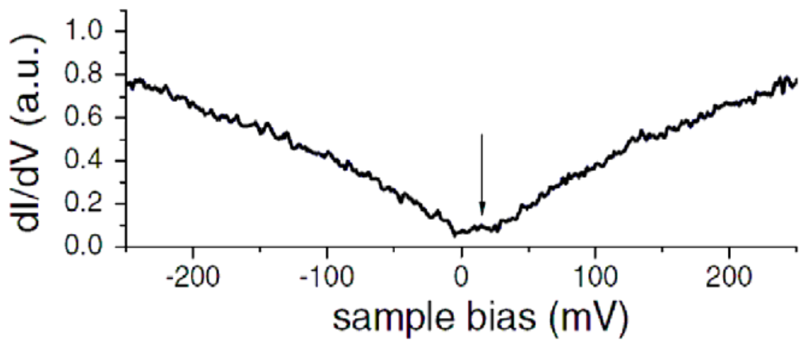

antenna ***a*** 6 rows apart from the boundary



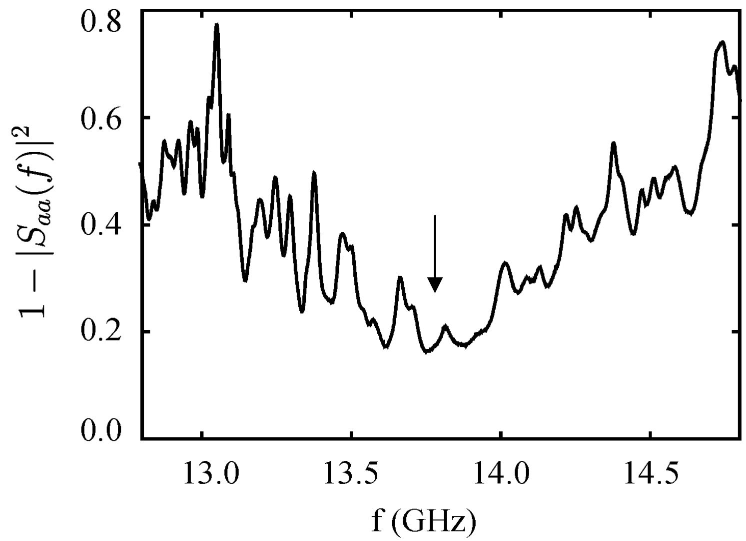

- Experimental Dirac frequencies agree with calculated one, $f_D$ =13.81 GHz, within the standard error of the fit
- Oscillations around the mean intensity → origin?

# Comparison with STM Measurements

graphene flake, Li *et al.* (2009)




photonic crystal



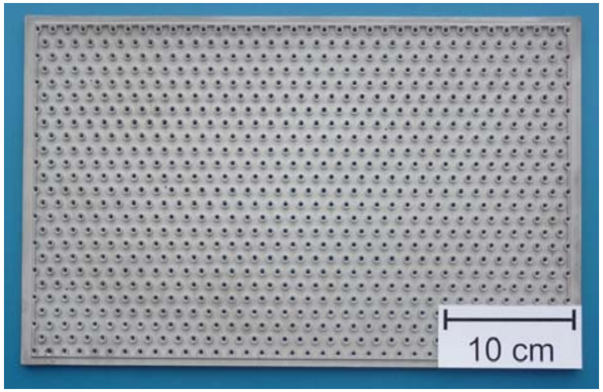

- Tunneling conductance is proportional to LDOS
- Similarity with measured reflection spectrum of the photonic crystal
- Oscillations in STM are not as pronounced due to the large sample size
- Finestructure in the photonic crystal shows fluctuations (RMT)

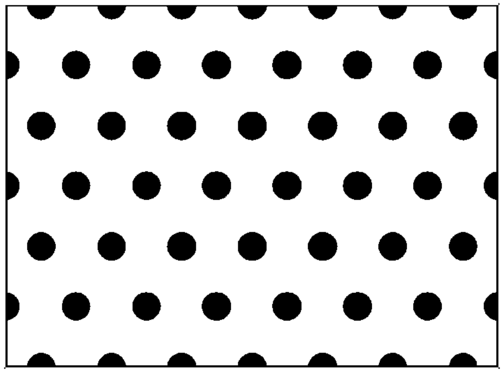
# Summary I

- Connection between reflection spectra and LDOS is established
- Cusp structure in the reflection spectra is identified with the Dirac point
- Photonic crystal simulates one particle properties of graphene
- Results are published in Phys. Rev. B **82** 014301 (2010)
- Measured also transmission near the Dirac Point
- Dirac billiards

# Dirac Billiard

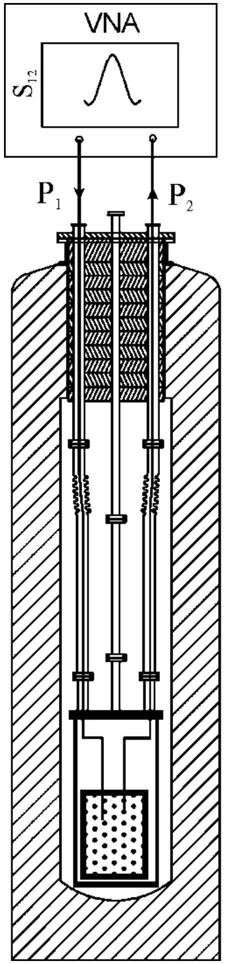
- Photonic crystal $\rightarrow$ box: bounded area = billiard

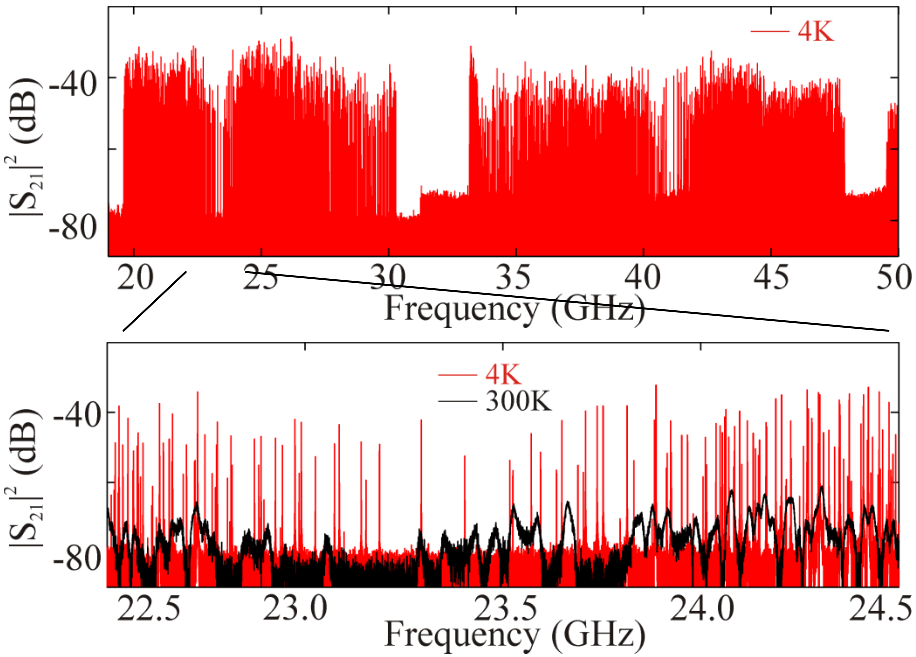


- 888 cylinders (scatterers) milled out of a brass plate
- Height $d$ = 3 mm $\rightarrow f_{max}^{2D}$ = 50 GHz for 2D system
- Lead plated $\rightarrow$ superconducting below 7.2 K $\rightarrow$ high Q value
- Boundary does not violate the translation symmetry $\rightarrow$ no edge states
- Relativistic massless spin-one half particles in a billiard
  (Berry and Mondragon,1987)

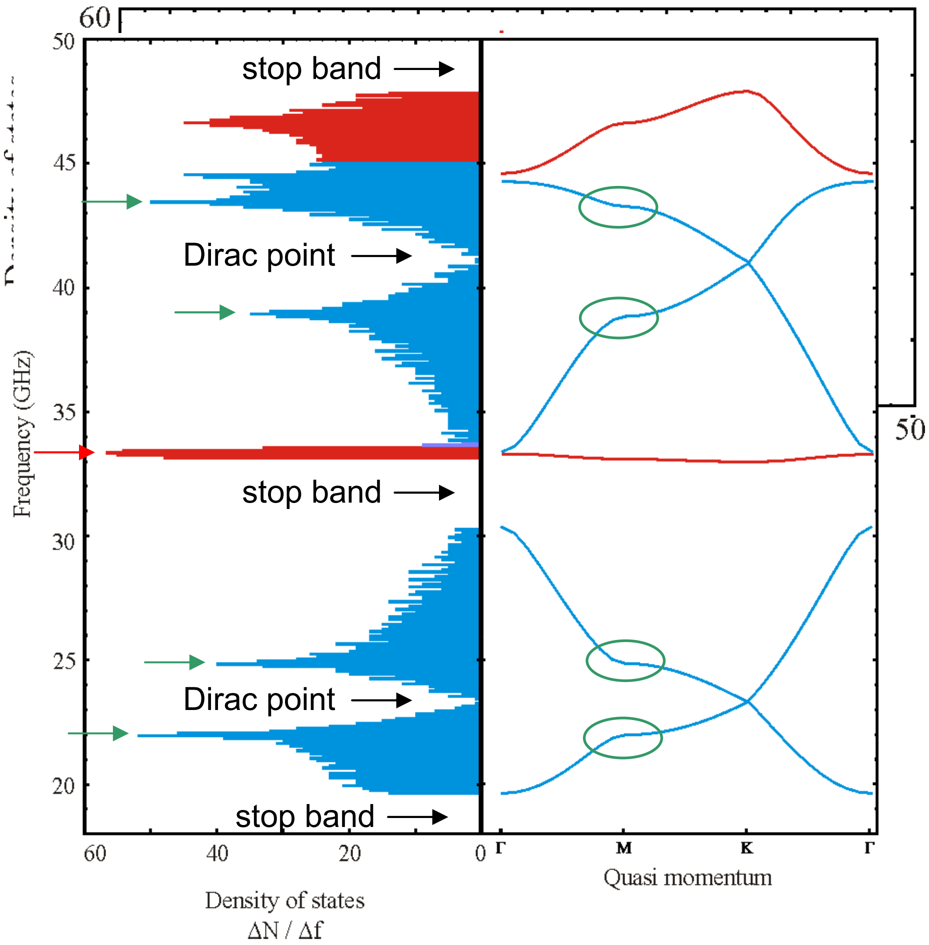
# Transmission Spectrum at 4 K




- Pronounced stop bands
- Quality factors > $5 \cdot 10^5$
- $\langle \Gamma \rangle / \langle D \rangle = 10^{-3} \rightarrow$ complete spectrum
- Altogether 5000 resonances observed

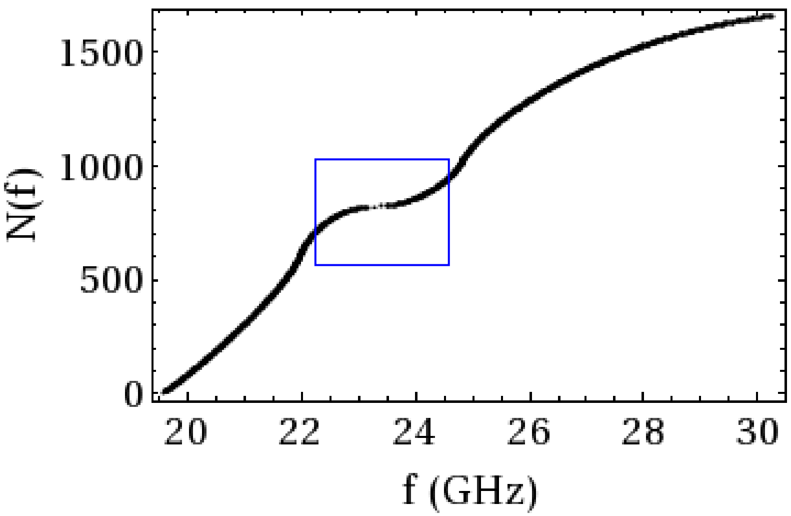
# Density of States of the Measured Spectrum and the Band Structure

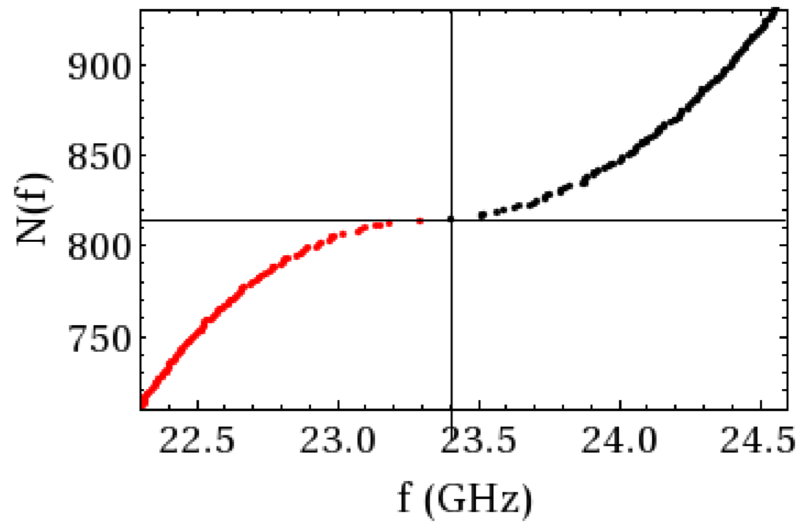



- Positions of stop bands are in agreement with calculation
- DOS related to slope of a band
- Dips correspond to Dirac points
- High DOS at van Hove singularities → ESQPT?
- Flat band has very high DOS
- Qualitatively in good agreement with prediction for graphene

(Castro Neto *et al.*, RMP **81**,109 (2009))

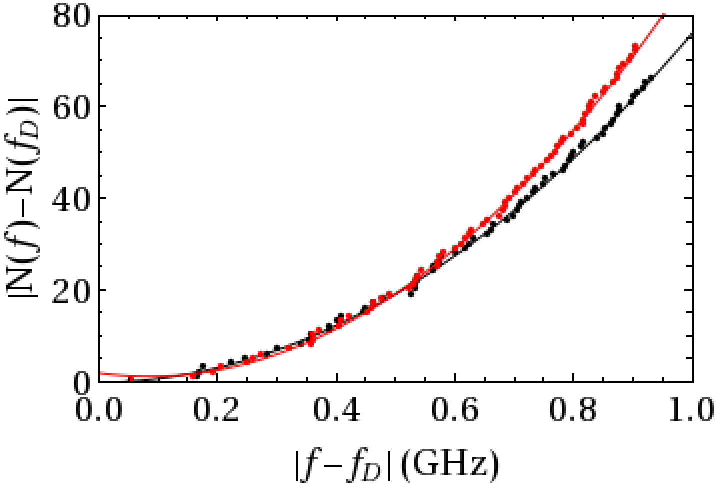
# Integrated Density of States: 1st and 2nd Bands

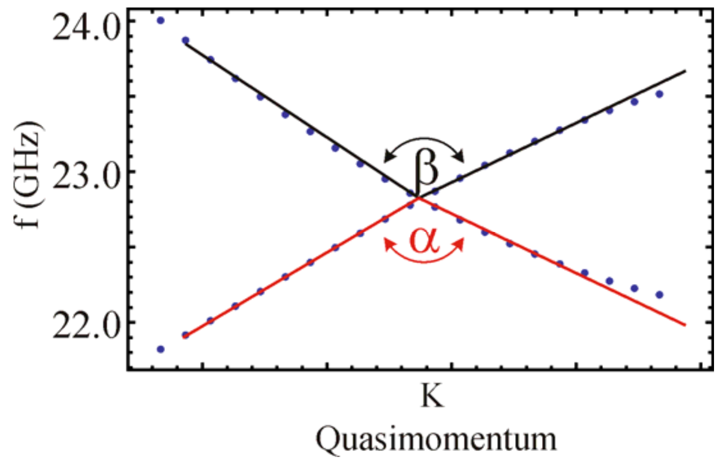


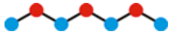




- Does not follow Weyl law for 2D resonators ($N_{Weyl}(f) = \frac{4\pi A}{c^2} f^2$)
- Small slope at the Dirac frequency $\rightarrow$ DOS nearly zero
- Nearly symmetric with respect to the Dirac frequency
- Two parabolic branches

# Integrated DOS near Dirac Point

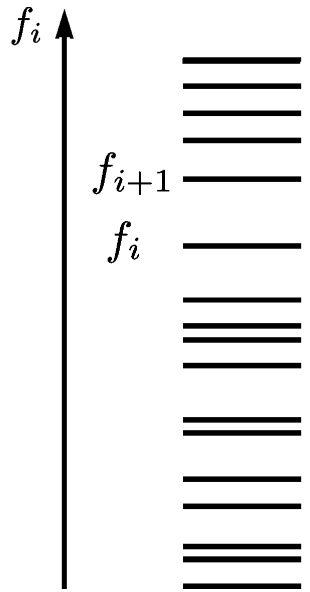


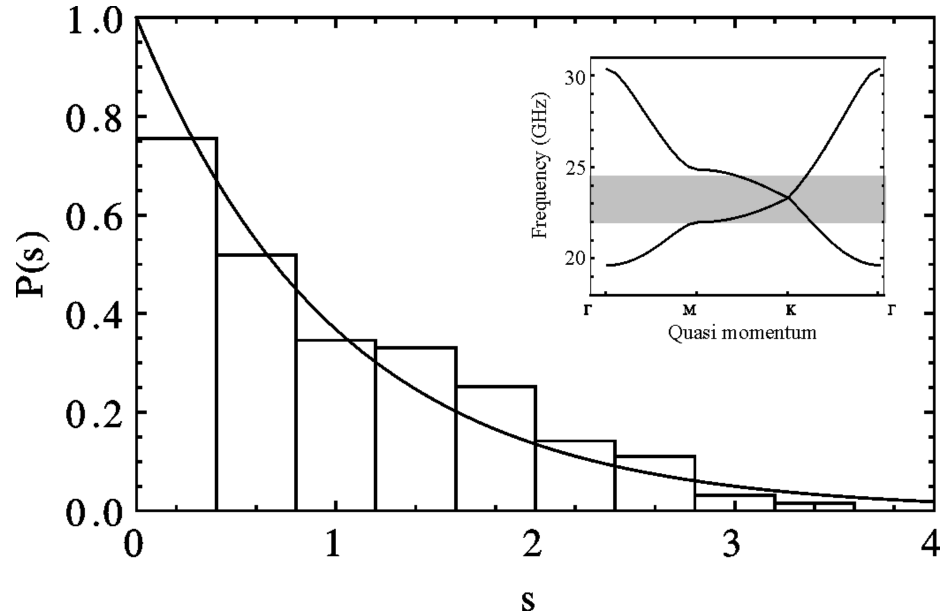

- Weyl law for Dirac billiard $N(k) = \frac{A}{2\pi}k^2 + \frac{U_{zz}}{\pi}k + C$ (J. Wurm *et al.*, PRB **84**, 075468 (2011))
  - $U_{zz}$ is length of zigzag edges 
  - $k = 2\pi\frac{|f - f_D|}{v_D}$
  - group velocity $v_D$ is a free parameter
- Same area $A$ for two branches, but different group velocities $\rightarrow$ α≠β

  $\rightarrow$ electron-hole asymmetry like in graphene

# Spectral Properties of a Rectangular Dirac Billiard: Nearest Neighbour Spacing Distribution

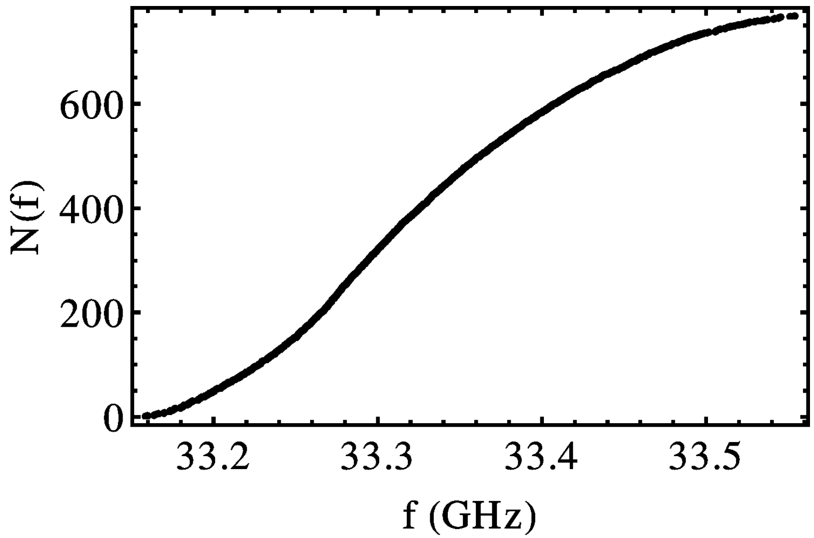


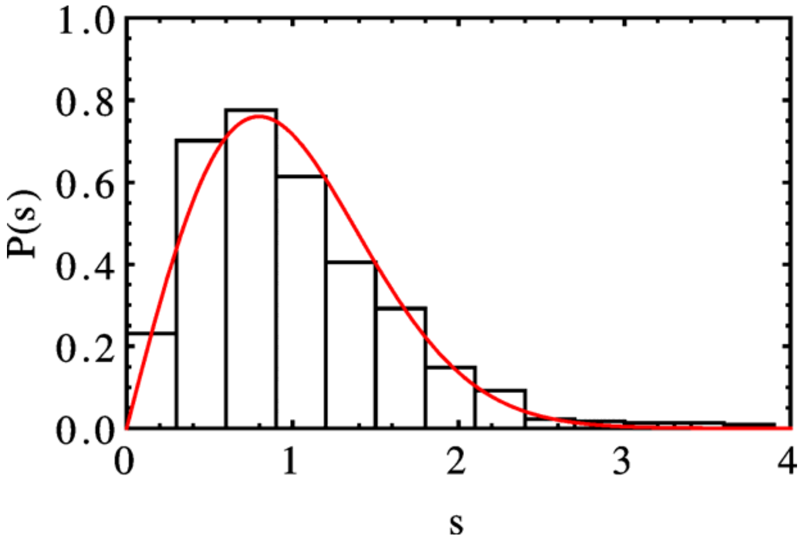

$$s_i = f_{i+1} - f_i$$




- 159 levels around Dirac point
- Rescaled resonance frequencies such that $\langle s_i \rangle = 1$
- Poisson statistics
- Similar behavior at second Dirac point

# NND: 3rd Band




- Very dense spectrum $\langle\Gamma\rangle/\langle D\rangle \approx 10^{-1}$
- Unfolding with a polynom of 8th order
- Missing levels?
- Seems to agree with GOE

# Summary II

- Photonic crystal simulates one particle properties of graphene
- Observation of edge states in the Dirac billiard (not shown)
- Realisation of superconducting microwave Dirac billiard i.e. photonic crystal in a metallic box serves as a model for a relativistic quantum billiard
- Experimental DOS agrees with calculated photonic band structure
- Fluctuation properties of the spectrum were investigated
- Open problems:
  - (i) Length spectrum of periodic orbits
  - (ii) Do we see an excited state quantum phase transition?

# Excited State Quantum Phase Transitions

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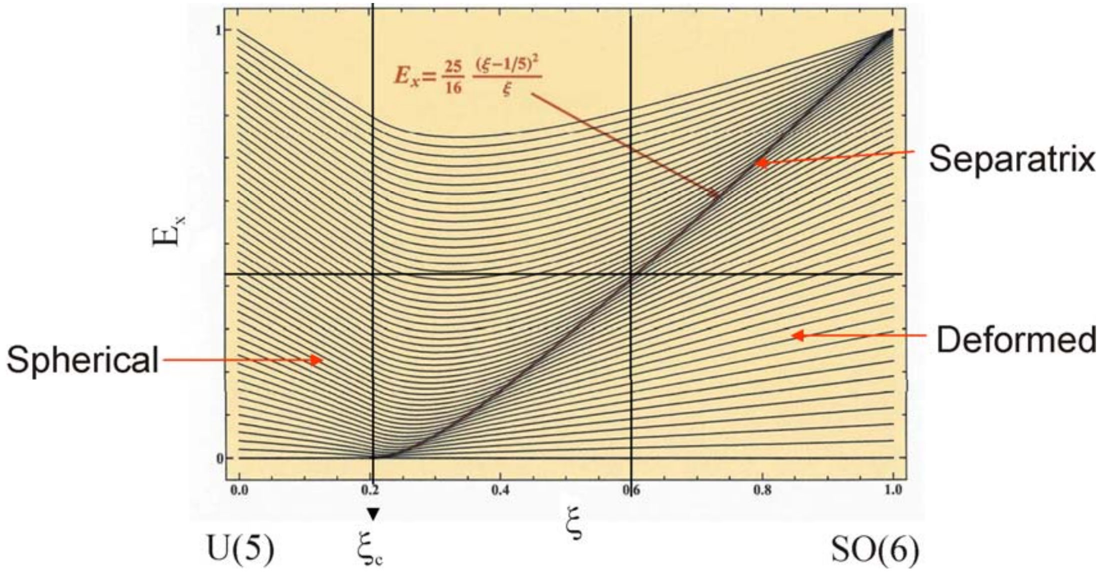
- Francesco Iachello at the “6th International Workshop in Shape-Phase Transitions and Critical-Point Phenomena in Nuclei” Darmstadt 2012




- Control parameter ξ
- At the separatrix the density of states diverges (Caprio, Cejnar, Iachello,2008)



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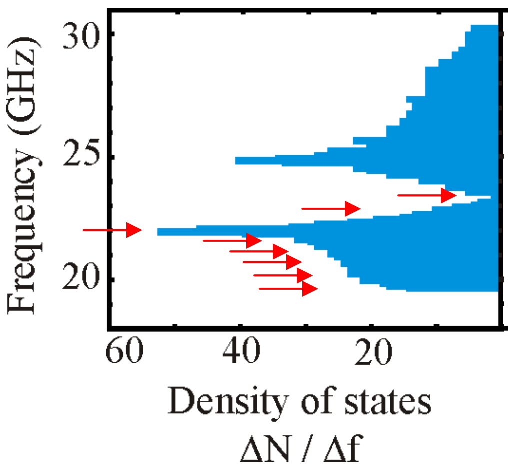
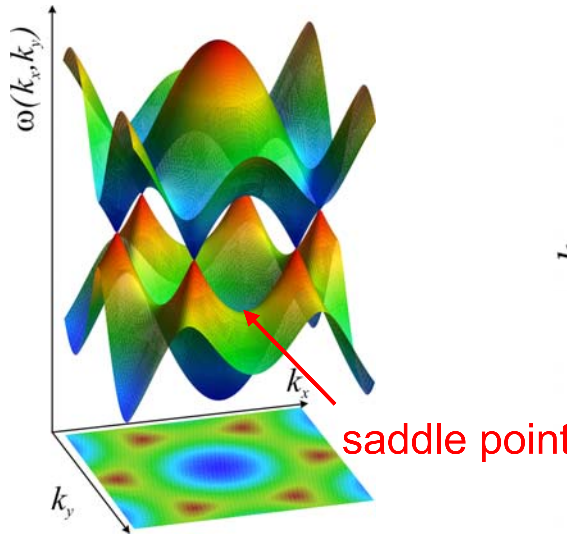
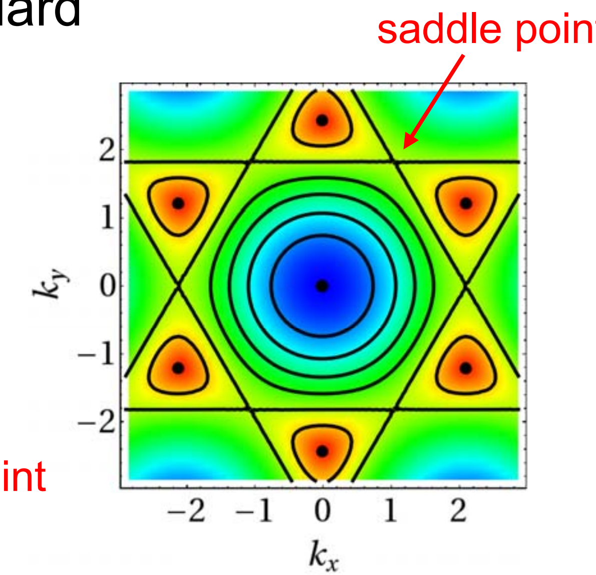
# Excited State Quantum Phase Transition in Dirac Billiard

- Experimental density of states in the Dirac billiard




- Van Hove singularities at saddle point: density of states diverges at $k$=M
- Possible control parameters
  - chemical potential
  - sublattice dependent potential
  - …

## Personal Remarks



.... The mode has no
several deformed nuclei, with
closer to those predicted in R
the subject of several experi
investigations.[7] The purpos

e and F. Palumbo, Phys. Rev. Lett. **41**,
and Nucl. Phys. **A326**, 193 (1979);
chi, F. Palumbo, and N. LoIudice, Phys.
96 (1984).
and D. J. Rowe, Nucl. Phys. **A289**, 461
Nucl. Phys. **A358**, 89c (1981); A. E. L.
g. Part. Nucl. Phys. **9**, 121 (1983).
T. Otsuka, F. Iachello, and I. Talmi, Phys.
(1977); T. Otsuka, A. Arima, F. Iachello,
hys. Lett. **76B**, 139 (1978).
. Richter, W. Steffen, A. E. L. Dieperink,
F. Palumbo, and O. Scholten, Phys. Lett.
).

638 citations

All the best and many more fruitful years together!

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Knüpfer and Bernard Metsch for several discussions,