Quantum Mechanics

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First Midterm Exam

Notice: in the following, $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ are unit vectors, while $\hat{x}$, $\hat{y}$, $\hat{z}$ are operators.

1. (45 points) A beam of spin-2 particles in the state $m_s = 2$ enters a Stern-Gerlach apparatus oriented along the z-axis.

a) (5 points) How many beams come out of the apparatus? What is their intensity in terms of the incoming intensity I_0?

b) (15 points) The apparatus is now rotated around the x-axis by an angle $\alpha \ll 1$. If the sensitivity of the output detectors is of order $\alpha^2 I_0$, how many beams will be detected? What are the eigenvalues of S_z for each of them?

c) (25 points) What are the intensities of the outgoing beams, to order α^2?

2. (40 points) A rotating body, consisting of two spin-1 particles in a bound state, is described by the hamiltonian $\hat{H} = \rho \hat{J}^2$, where $\hat{J}$ is the angular momentum operator for the body. It is known that the system is in the state $|\alpha\rangle = |m_1 = 1,\, m_2 = 0\rangle$.

a) (10 points) What are the possible outcomes of a measurement of the particle's rotational energy?

b) (15 points) Is $|\alpha\rangle$ a hamiltonian eigenstate? What is the expectation value of $\hat{H}$ in the state $|\alpha\rangle$?

c) (15 points) A beam of such particles, all in the state $|\alpha\rangle$, is passed through a Stern-Gerlach apparatus with its axis along $\hat{\mathbf{z}}$. How many beams emerge? What is the ratio of their intensities?

3. (65 points) A three-dimensional harmonic oscillator is described by the hamiltonian

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + \frac{1}{2} m r^2 \,, \tag{1}$$

where $r \equiv |\mathbf{r}|$ and $\mathbf{r}$ is the particle's position vector.

a) (5 points) What are the four lowest-lying eigenstates of $\hat{H}$ and what are their energies? Is there any degeneracy?

b) (15 points) Given the form of (1), what can you say about the angular part of the energy eigenfunctions? Write out explicitly the wavefunction for the ground state. How does it transform under arbitrary rotations? Is it an eigenstate of the operators $\hat{L}^2$, $\hat{L}_z$? What are their expectation values?

c) (15 points) Express the orbital angular momentum generators $\hat{L}_i$, $i = x, y, z$, in terms of the creation-annihilation operators of the oscillator.

d) (15 points) Consider the triplet of states

$$-\sqrt{\frac{3}{8\pi}}(-\hat{x} + i\hat{y})|\Omega\rangle \,, \qquad -\sqrt{\frac{3}{4\pi}}\hat{z}|\Omega\rangle \,, \qquad \sqrt{\frac{3}{8\pi}}(\hat{x} - i\hat{y})|\Omega\rangle \,, \tag{2}$$

where $|\Omega\rangle$ is the ground state. What is the expectation value of $\hat{L}_i$, $\mathbf{L}^2$ in each of them?

e) (15 points) A perturbation $\hat{V} = \lambda \mathbf{B} \cdot \mathbf{L}$ is turned on, with $\mathbf{B} = B_0 \hat{\mathbf{z}}$. What is the resulting first-order spectrum for the above four states? Make a graph depicting the new energy levels.

4. (20 points) Problem 37 from Chapter 5 of Sakurai.

5. (30 points) A particle of mass m is confined in an one-dimensional infinite potential well extending from $x = 0$ to $x = L$. At $t = 0$, the particle is in the second excited state and a perturbation $V(t) = 2A\delta(x - \frac{L}{2}) \cos^2 \omega t$ is turned on.

a) (10 points) Which are the final states the particle *cannot* jump into, regardless of the value of ω?

b) (10 points) Are there any values of ω for which the particle can jump into the ground state? For these values of ω, can the particle jump into any other state?

c) (10 points) What is the transition rate to the ground state in the case of item (b)?